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Two-Dimensional Steady Flow of Stably Stratified Incompessibile Inviscid Towards a Sink-Investigation of Role of Stream Function and Velocity

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ABSTRACT

Dube & Kumari (2023) work is further taken up to investigate the role of stream function ψ , Pseudo stream function ψ' and velocity v (vertical and horizontal components). They considered the problem of steady two-dimensional flow of a stably stratified, incompressible inviscid fluid towards a sink situated at the middle point (origin) of the lower horizontal boundary of an infinite channel. They studied the flow pattern only. The problem is investigated by the physical flow to a pseudo-flow as Yih (1958) and Dube (2002) did in the semi-infinite channel problem. The solution of the pseudo-flow is obtained first by the linearizing its equation and then by using the Fourier transforms.

The pseudo-flow is characterized by one parameter β , which is the inverse of the internal Froude number F. When $\beta = 0$, the stratified fluid flow reduces to the homogeneous irrotational fluid flow (unaffected directly by gravity).

Their solution for β for one-half of the channel agrees well with that of Yih and Dube. But when $\beta \neq 0$, our solution differs greatly from Yih and Dube . It is found that as soon as β becomes non-zero, a central core in the form of α cone not occupied by the fluid starts developing around the vertical line through the sink. The top of the core becomes wider and wider as β goes on increasing towards π . The reason for the development of core may be attributed to gravity effect.

Nomenclature:

F	:	Froude number
F_0	:	Ordinary Froude number
$\underline{q}$	:	Velocity vector
$\underline{q}'$	:	Dimension of velocity of pseudo velocity
$\underline{\underline{F}}$	:	External force other than gravitational force
$\underline{g}$	:	Acceleration due to gravity
p	:	Pressure
ρ	:	Density
ρ_0	:	Reference density
U	:	Horizontal velocity
U_0	:	Representative velocity
d	:	Reference length
g	:	Acceleration due to gravitation
ψ	:	Stream function
ψ'	:	Pseudo stream function
$\underline{k}$	:	Unit vector perpendicular to the plane of the motion
δ	:	Variation of height of the streamline.

FORMULATION AND SOLUTION:

Dube & Kumari (2023) got the velocity as

$$v = -2\pi \sum_{n=1}^{\infty} \frac{n \sin \pi n y}{(n^2\pi^2 - \beta^2)^{1/2}} \exp\left[-lxl(n^2\pi^2 - \beta^2)^{1/2}\right].$$

The solution given by the above equation is valid for the entire region of flow. The solution shows that v can never be positive for all x and $0 \le y \le 1$.

Now since v is known, ψ can be determined by integrating $v = -\frac{\partial\psi}{\partial x}$, and then u is determined from $u = \frac{\partial\psi}{\partial y}$.

Integrating the equation $\left(\nabla^2 + \beta^2\right)\psi = -\beta^2 y, (x > 0)$ and $\left(\nabla^2 + \beta^2\right)\psi = \beta^2 y, (x < 0)$ with respect to x and using the boundary condition that $\psi_+ = -y$ at $x = \infty$, we get (since $A = -2$)

$$\psi_+ = -y - 2\pi \sum_{n=1}^{\infty} \frac{n \sin n\pi y}{\left(n^2\pi^2 - \beta^2\right)} \exp\left[-x(n^2\pi^2 - \beta^2)^{1/2}\right] \text{ for } x > 0 \qquad (1)$$

where ψ_+ is used to denote the pseudo-stream function for the right half of the channel (i.e. for $x > 0$).

Similarly, if ψ_- denotes the pseudo-stream function for the half of the channel (i.e. for $x < 0$), we get

$$\psi_- = y + 2\pi \sum_{n=1}^{\infty} \frac{n \sin n\pi y}{\left(n^2\pi^2 - \beta^2\right)} \exp\left[x(n^2\pi^2 - \beta^2)^{1/2}\right] \text{for } x < 0. \quad (2)$$

The equations (1) and (2) describe the pseudo-flow on the right and the left half of the channel respectively.

It may be noted that our solution for ψ will lead to the Yih's solution if β is removed from the denominator.

Further our solution for ψ also show that unlike Yih's solution, when $\beta = \frac{1}{n\pi}$ $(n = 1, 2, 3, \ldots)$ the stream function does not exist. Hence the values $F = \frac{1}{n\pi}$ $(n = 1, 2, 3, \ldots)$ seem to be the critical values of the Froude number for the problem. In fact Debler (1959) pointed that $F = \frac{1}{\pi}$ is critical Froude number for the semi-infinite problem.

If u_+ and u_- denote the horizontal components of pseudo-velocity for the right $(x > 0)$ and the left $(x < 0)$ half of the channel respectively, then differentiating the equations (1) and (2) partially with respect to y, we get

$$u_+ = -1 - 2\pi^2 \sum_{n=1}^{\infty} \frac{n \cos n\pi y}{\left(n^2\pi^2 - \beta^2\right)} \exp\left[-x\left(n^2\pi^2 - \beta^2\right)^{1/2}\right] \text{ for } x > 0 \qquad (3)$$

and

$$u_- = 1 + 2\pi^2 \sum_{n=1}^{\infty} \frac{n \cos n\pi y}{\left(n^2\pi^2 - \beta^2\right)} \exp\left[x\left(n^2\pi^2 - \beta^2\right)^{1/2}\right] \text{ for } x < 0 . \qquad (4)$$

It can now be easily seen from the equations (3) and (4) that u_- is the mirror image of u_+ with respect to the $y-$ axis.

We now examine the behaviour of u near the point $x = 0, y = 1$ for small β .Considering for $x > 0$, the equation (3) can be written as (putting $\theta = 1 - y$ and simplifying)

$$u_+ = -1 - 2\left[\begin{array}{l} \frac{1}{2}\left(1 - \frac{\sinh \pi x}{\cosh \pi x + \cos \pi\theta}\right) + \frac{\theta^2}{4\pi}\left\{\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}\exp(-n\pi x) - \pi \tan^{-1}\frac{\sin \pi\theta}{\exp(\pi x) + \cos \pi\theta} + O(\theta^2)\right\} \\ + \frac{\theta^2}{4\pi} x \log\left\{1 - \exp(-2\pi x) + 2\exp(-\pi x)\cos \pi\theta\right\} + O(\beta^4) \end{array}\right]$$

Therefore when θ is small, the above equation gives

$$u_+ = -1 + 2\left[\frac{1}{2}\left(1 - \tanh\frac{\pi x}{2}\right) + \frac{\beta^2}{\pi^2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}\exp(-n\pi x) + \frac{\beta^2}{2\pi}\log\{1 + \exp(-\pi + x)\}\right] + O(\beta^4) .$$

For small x, the above expression further gives

$$u_+ = \frac{\beta^2}{6} - \frac{\pi}{2}\left(1 - 2\frac{\beta^2}{\pi^2}\log 2\right)x + O(x^2) + O(\beta^4) \qquad (5)$$

This is the expression for u near the upper corner of the right half of the channel.

Similarly for the left half of the channel, we find near the upper corner

$$u_- = -\frac{\beta^2}{6} - \frac{\pi}{2}\left(1 - 2\frac{\beta^2}{\pi^2}\log 2\right)x + O(x^2) + O(\beta^4) . \qquad (6)$$

From the equations (5) and (6) , it is seen that when x is sufficiently small $u_+ > 0$ and $u_- < 0$.

But far away the y-axis (i.e. when x is not small) $u_+ < 0$ and $u_- > 0$ (as the flow is towards the sink).

So, at the same level of y near the upper boundary, u (the horizontal velocity) becomes zero before the fluid particle reaches to the y -axis. Therefore on the upper boundary, $u = 0$ (for small β) at the points $x \cong \pm \frac{\beta^2}{3\pi}$, which are obtained from the equations (5) and (6).

Hence a fluid particle lying particularly near the upper boundary (i.e. $y = 1$) will have , in course of its motion from infinity, zero horizontal velocity before it can reach the y -axis. This confirms our earlier assertion that the fluid can't come in contact with the y -axis except converging at the origin.

Since the fluid on both sides of the y -axis can't come in contact with the y -axis , a central core not occupied by the fluid has developed around the y -axis. This core is bounded on the left and the right by the parts of the streamline $\psi_- = 1$ and $\psi_+ = -1$ from the vanishing points of u on the upper boundary to the origin.

If ζ denotes the velocity corresponding to the pseudo-low, the equation (2.2.6) and (2.2.5) give

$$\zeta = -\nabla^2\psi = \beta^2(\psi_+ + y) \text{ , for } x > 0$$

$$= \beta^2(\psi_- - y) \text{ , for } x < 0 \qquad (7)$$

On using the expressions for ψ_+ and ψ_- from the equations (1) and (2) , we get

$$\zeta_+ = 2\pi\beta^2 \sum_{n=1} \frac{n \sin n\pi x}{n^2\pi^2 - \beta^2} \exp\left[-x(n^2\pi^2 - \beta^2)^{1/2}\right] \text{ , for } x > 0$$

$$\zeta_- = -2\pi\beta^2 \sum_{n=1} \frac{n \sin n\pi x}{n^2\pi^2 - \beta^2} \exp\left[x(n^2\pi^2 - \beta^2)^{1/2}\right] \text{ , for } x < 0 \qquad (8)$$

These are the vorticity expressions corresponding to the pseudo-flow for the right and left half of the channel. From equation (7), it is evident that β is directly responsible for the pseudo-vorticity. The equation also shows that for fixed β , the magnitude of the vorticity depends on the deviation of the streamline from the uniform stream at infinity. The vorticity is not zero anywhere except at infinity and at the origin and also at the upper and lower boundaries.

Al core not occupied by the fluid develops around the y -axis above the sink. The core is bounded on the left and the right by the parts of the uppermost streamline from vanishing points of u (on the upper boundary) to the origin (sink). We shall now have a short pressure discussion to support that the core can't be occupied by the fluid even with the lowest density ρ_1 .

The pressure equation is dimensional quantities for the pseudo-flow is the equation

$$H = p + \frac{1}{2}\rho_0 q'^2 + \rho g y \qquad (9)$$

when non-dimensionalized by measuring H, p in terms of $\rho_0 U_0^2$ and q', ρ, y in terms of U_0, ρ_0, d respectively. The Equation (9) becomes

$$H = p + \frac{1}{2}q^2 + \rho\frac{\beta^2}{\alpha}y \qquad (10)$$

This H is constant along a streamline.

At infinity $\frac{dH}{D\psi} = -\beta^2$.

Integrating it, we have $H = -\frac{1}{2}\beta^2\psi^2 + C$ (11)

where C is a constant.

Since H is a constant along a streamline, therefore the equations (10) and (11) give (corresponding to a streamline)

$$-\frac{1}{2}\beta^2\psi^2 + C = p + \frac{1}{2}q^2 + \frac{\beta^2}{\alpha}\rho y . \qquad (12)$$

Hence, at the point of height y on the either side of the left and the right boundaries of the core, we have

$$-\frac{1}{2}\beta^2 + C = p + \frac{1}{2}q^2 + \frac{\beta^2}{\alpha}(1-\alpha)y \qquad (13)$$

(as on the left boundary, $\psi = 1$ and on the right boundary $\psi = -1$, and as also the streamline $\psi = \pm 1$, ρ is constant having the value $(1-\alpha)$).

But, if p_∞ be the pressure at infinity ($|x| = \infty$) on the streamline $\psi = 1$ or $\psi = -1$, then

$$C = \frac{1}{2}\beta^2 = p_\infty + \frac{1}{2\alpha}\beta^2(1-\alpha) . \qquad (14)$$

Therefore, from the equations (13) and (14) , we have

$$p = p_\infty + \frac{1}{2}(1-q^2) + \frac{1}{\alpha}\beta^2(1-\alpha)(1-y). \qquad (15)$$

This gives the non-dimensional pressure at a point of height y on either side of left and the right boundaries of the core.

The term $\frac{1}{\alpha}\beta^2(1-\alpha)(1-y)$ in the equation (15) is nothing but the non-dimensional hydrostatic pressure p_H (at the point considered) of a fluid with the non-dimensional lowest density $(1-\alpha)$, so the equation (15) can be written as

$$p = p_\infty + \frac{1}{2}(1-q^2) + P_H .$$

Therefore, $p < P_H$ if $q^2 > 1 + 2p_\infty$.

Near the sink, q is large (as it is $O\left(\frac{1}{r}\right)$), and p is finite, and so $q^2 > 1+2p_\infty$ near the sink. Therefore, $p < P_H$ near the sink.

This implies that the pressure near the sink on either side of the left and right boundaries of the core can't maintain a vertical column of fluid even with the lowest density. It therefore follows that the core bounded on left and right respectively by the parts of the streamlines $\psi = 1$ and $\psi = -1$ can't be occupied by the fluid even with the lowest density. As air also can't be expected to enter into the core, the portion is somewhat similar to the torricellian vacuum.

The pseudo-flow is obtained from the actual flow through the transformation

$$\bar{q} = \left(\frac{\rho}{\rho_0}\right)^{-1/2} \bar{q}' . \qquad (16)$$

In this relation, $\bar{q}$ is the velocity in actual flow, $\bar{q}'$ the velocity in the pseudo-flow, ρ is the density and ρ_0 is the density at the lower boundary of the channel.

When the velocities are made non-dimensional, the above relation is not changed the form.

Now, writing $\bar{q}_a$ and $\bar{q}$ for the non-dimensional velocities in the actual and the pseudo-flows respectively. we have

$$\bar{q}_a = \left(\frac{\rho}{\rho_0}\right)^{-1/2} \bar{q} . \qquad (17)$$

For the left half of the channel (i.e. for $x < 0$), this gives

$$\bar{q}_a = \left(1-\alpha\psi_-\right)^{-1/2} \bar{q} . \qquad (18)$$

Similarly for the right half of the channel (i.e. for $x > 0$)

$$\bar{q}_a = \left(1-\alpha\psi_+\right)^{-1/2} \bar{q} \qquad (19)$$

where ψ_+ is given by the equation (1).

The equations (18) and (19) give the velocity distributions of the actual flow through the left and the right half of the channel.

From the equation (17), we have $\frac{|\bar{q}_a|}{|\bar{q}|} = \left(\frac{\rho}{\rho_0}\right)^{-1/2} \geq 1$ as $\rho < \rho_0$.

The implies that the velocity in the actual flow is magnified by the factor $\left(\frac{\rho}{\rho_0}\right)^{-1/2}$. This is, in fact, a consequence of the transformation given by the equation (16).

To obtain the stream function, if ψ_a, ψ denote the stream function in the actual and the pseudo-flows respectively, then from equation (17), we have

$$d\psi_a = \left(\frac{\rho}{\rho_0}\right)^{-1/2} d\psi \text{ , whence } \psi_a = \int \left(\frac{\rho}{\rho_0}\right)^{-1/2} d\psi \ .$$

Hence for the left half of the channel $\psi_a = \int (1-\alpha\psi_-)^{-1/2} d\psi_- = \frac{2}{\alpha}\left[1-(1-\alpha\psi_-)^{1/2}\right]$. (20)

(as ψ_a is not zero with ψ)

Similarly for the right half of the channel

$$\psi_a = \frac{2}{\alpha}\left[(1-\alpha\psi_+)^{1/2}-1\right]. \qquad (21)$$

In the equation (20), ψ_- is given by the equation (2) while in equation (21), ψ_+ is given by equation (1).

It follows from the equations (20) and (21) that the velocity of the actual flow (i.e. $\nabla^2\psi_a$) is , unlike in the pseudo-low, not zero at infinity (i.e. $x = \pm\infty$).

It may also be noted that the actual flow is affected directly (and independently) by the stratification α .

So far we have considered by the restricting to the case when the internal Froude number $F > \frac{1}{\pi}$, i.e. $\beta < \pi$. Now we consider the case when $\beta > \pi$.

The solution for ψ, u, v (equations (1),(2),(3) ,(4) and the equation

$$v = A\sum_{n=1} \frac{n\sin n\pi y}{(n^2\pi^2-\beta^2)^{1/2}} \exp\left[-|x|(n^2\pi^2-\beta^2)^{1/2}\right]$$

are valid only for $\beta < \pi$. These solutions show that they do not exist when $\beta = n\pi$, i.e. $F > \frac{1}{n\pi}$ ($n = 1,2,3,...$). We shall now consider the solution, if possible, when $\beta > \pi$ but $\neq n\pi$ ($n = 1,2,3,...$).

We rewrite the solution of the equation $\left(\alpha^2+\beta^2\right)v = 0$ in the integral form as

$$v = -\frac{1}{\pi}\int_{-\infty+i0}^{\infty+i0} \frac{\sinh\left[(t^2-\beta^2)^{1/2}(1-y)\right]}{\sinh(t^2-\beta^2)^{1/2}} \exp(itx)dt \ . \qquad (22)$$

We shall evaluate the integral appearing on the right hand side of the equation (22) for the value of the parameter β such that $(N-1)\pi < \beta < N\pi$, N being a positive integer ≥ 2 ; and to do this we first note that $\int_{-\infty+io}^{\infty+i0} = \int_{-\infty}^{\infty}$ and then we employ the same contour integration method to the later as employed in the case when $\beta < \pi$.

When $(N-1)\pi < \beta < N\pi$, the poles of the function $\frac{\sinh\left[(t^2-\beta^2)^{1/2}(1-y)\right]}{\sinh(t^2-\beta^2)^{1/2}}\exp(itx)$ are

$$t = \pm(\beta^2 - n^2\pi^2)^{1/2}, n = 1,2,3,...,(N-1)$$

and $\quad t=\pm i(n^2\pi^2-\beta^2)^{1/2}, n=N,(N+1),(N+2),\ldots\infty$.

Hence, taking the same contour C as earlier (evaluation of integral for $\beta<\pi$), but indented(by small semi-circles) at the poles on the real axis , we get

$$\int \frac{\sinh\left[(t^2-\beta^2)^{1/2}(1-y)\right]}{\sinh(t^2-\beta^2)^{1/2}}\exp(itx)dt = 2i\pi\begin{bmatrix}\frac{1}{2}\text{ sum of the residues of poles on C +}\\ \text{Sum of residues at imaginary poles within C}\end{bmatrix}$$

$$=2\pi^2\sum_{n=1}^{N-1}\frac{n\sin n\pi y}{(\beta^2-n^2\pi^2)^{1/2}}\sin\left[x(\beta^2-n^2\pi^2)^{1/2}\right]+2\pi^2\sum_{n=1}^{N-1}\frac{n\sin n\pi y}{(n^2\pi^2-\beta^2)^{1/2}}\exp\left[-x(n^2\pi^2-\beta^2)^{1/2}\right]$$

So the solution for v for the right half of the channel ,i.e. for $x>0$ is given by

$$v_+=-2\pi\sum_{n=1}^{N-1}\frac{n\sin n\pi y}{(\beta^2-n^2\pi^2)^{1/2}}\sin\{x(\beta^2-n^2\pi^2)^{1/2}\}+\sum_{n=N}^{\infty}\frac{n\sin n\pi y}{(n^2\pi^2-\beta^2)^{1/2}}\exp\{-x(n^2\pi^2-\beta^2)^{1/2}\}, \quad (23)$$

and similarly for the left of the channel i.e. for $x<0$ is given by

$$v_-=-2\pi\sum_{n=1}^{N-1}\frac{n\sin n\pi y}{(\beta^2-n^2\pi^2)^{1/2}}\{\sin x(\beta^2-n^2\pi^2)^{1/2}\}+\sum_{n=N}^{\infty}\frac{n\sin n\pi y}{(n^2\pi^2-\beta^2)^{1/2}}\exp\{x(n^2\pi^2-\beta^2)^{1/2}\}. \quad (24)$$

In each of the two equations (23) and (24) , the first part involving the summation $\sum_{n=1}^{N-1}$ represents indeed a wave. So when $\beta>\pi$ and $\neq n\pi$, upstream waves exist. That is really in conformity to the results shown by Yih and Dube in connection with the semi-infinite problem that if $F<\frac{1}{\pi}$ or $\beta>\pi$ upstream waves will occur.

But the upstream waves do not maintain the boundary conditions at infinity, i.e. the condition $v=0$ at infinity ($x=\pm\infty$) is violated. So, the upstream waves violate the boundary conditions at infinity. However, they do not violate the assumption that $\frac{d\rho}{d\psi}$ is constant at infinity (as the equation for v itself is derived on this assumption).

Hence, when $\beta>\pi$ and $\neq n\pi$, no solution (which does not violate all the conditions at infinity) is possible. Like Yih and Dube, we can simply say that when $\beta>\pi$, an exact solution of the equation $\nabla^2 v+\beta^2 v=0$ can still be obtained if the waves are allowed upstream and if $\frac{d\rho}{d\psi}$ is still constant, and that the solution is given by the equations (23) and (24).

But it is seen that the solutions do not exist for $\beta=n\pi$. Again, it is also seen that when $\beta>\pi$ and $\neq n\pi$, no solution consistent with the boundary conditions at infinity exists. Hence it is inferred that $\beta=\pi$ is a critical value of β for the problem, i.e. $F=\frac{1}{\pi}$ is the critical internal Froude number, as happened in the case of the semi-infinite problem.

Conclusion:

The pseudo-flow is a one parameter flow, i.e. to say the pseudo-flow has, occurring in it, only one parameter β which is the inverse of the internal Froude number $F = \left[\frac{U_o^2}{\alpha\, gd}\right]^{1/2}$.

The case $\beta = 0$ (implying the stratification $\alpha = 0$) corresponds to the homogeneous irrotational flow with density $\rho = \rho_0$ everywhere in the channel and unaffected directly by gravity.

In the case $\beta = 0$, the expressions of ψ (ref to equations (1) and (2)), u and v then reduce to

$$\psi_+ = -y - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\pi y\,.\exp(-n\pi x) = -y - \frac{2}{\pi}\tan^{-1}\left[\frac{\sin\pi y}{\exp(\pi x) - \cos(\pi y)}\right], \ (for\ x \ge 0)$$

$$\psi_- = y + \frac{2}{\pi}\tan^{-1}\left[\frac{\sin\pi y}{\exp(\pi x) - \cos(\pi y)}\right], \ (for\ x \le 0)$$

$$u_+ = u_- = -\frac{\sinh\pi x}{\cosh\pi x - \cos\pi y} \text{ and } v_+ = v_- = -\frac{\sinh\pi y}{\cosh\pi x - \cos\pi y}.$$

These give the stream functions and the velocity components in the homogeneous fluid flow. The above expressions for ψ_+ and ψ_- show that at $x = 0$.

$$\psi_+ = -y - \frac{2}{\pi}\tan^{-1}\left[\frac{\sin\pi y}{1 - \cos(\pi y)}\right] = -1,$$

Similarly $\psi_- = 1$.

Also at $x = 0, y = 1$, the fluid velocity is zero. But at the upper boundary $(y = 1)$ on the right of the channel, $\psi_+ = -1$ and similarly at the upper boundary on the left of the channel $\psi_- = 1$.

Hence it is concluded that the uppermost streamlines on the right and left of the channel reach the y-axis, and join along the y-axis from $y = 1$ to $y = 0$. This makes the part of the y-axis a streamline.

But on the y-axis, the stream function is discontinuous, the amount of discontinuity being $\psi_+ - \psi_- = -2$.

The flow in the channel is, no doubt, symmetrical about the y-axis. The streamlines patter in the channel for $\beta = 0$ is seen as in the figure. The figure-1 shows that the streamlines starting from infinity in the lower half of the channel become closer as one goes towards the y-axis, while the streamlines in the upper half become widened. This implies that the flow in the lower half of the channel become faster towards the y-axis while the flow in the upper half becomes slower. This can be verified by considering the expressions for the velocity components.

Again, near the point $x = 0, y = 1$(where the fluid velocity is zero), the motion on either side of the y-axis is very slow, and as a consequence; accumulation of the fluid occurs near that point, i.e. the point where the fluid velocity is zero.

When $\beta = 0$, but $< \pi$, the above feature still occurs with the difference that the uppermost streamline on the left and the right of the channel cannot reach the y-axis thereby allowing to form around the y-axis , a central core not occupied by fluid which is caused by the gravity effect.

When $\beta = 0$, the fluid velocity vanishes only at the point $x = 0, y = 1$, i.e. only one stagnation point appears at $x = 0, y = 1$. But $\beta \neq 0$, two stagnation point appear on the upper boundary (i.e. on $y = 1$), one on each side of the y-axis and equidistant from it.

This shows that the two stagnation points coalesce at one point ($x = 0, y = 1$) when $\beta = 0$, but start separating as soon as β becomes non-zero. The separation becomes wider and wider as β increases to π .

When $\beta = 0$, the uppermost streamlines (corresponding to $y = 1$) on the left and right of the y-axis meet at the common stagnation point on the y-axis, and join together forming one streamline up to the origin. But when $\beta \neq 0$ and also since there are two stagnation points on the upper boundary $y = 1$, there occurs a different picture. The upper most streamlines do not occurs a different picture. The uppermost streamlines do not cross the stagnation points to reach the y-axis, but instead, bend sharply downwards at those stagnation points and then go further down to join only at the origin (sink). This implies no fluid particle can come in contact with the y-axis except at the origin. Thus when $\beta \neq 0$, a torricellian vacuum like core bounded on left and right by parts of the uppermost streamlines $\psi_- = 1$ and $\psi_+ = -1$, and not occupied by the fluid has developed around the y-axis. The core takes a very interesting shape: it is wide at the top and narrow down to appoint at the origin. As β increases, the core becomes wider only at the top. This can be seen from the figures 2 and 3 , which are down to show the flow pattern from two different values of β namely $\beta = 1$ and $\beta = 2$.

Our solution for $\beta = 0$ for one-half of the channel agrees well with the Yih (1959) and Dube et.al. (2023). But when $\beta \neq 0$, our solution differs greatly from that of Yih and Dube at.al.

It is found that as soon as β becomes non-zero, a central core in the form of α cannot be occupied by the fluid starts developing around the vertical line through the sink. The top of the core becomes wider and wider as β goes on increasing towards π . The reason for the development of the core may be attributed to gravity effect.

When $\beta \geq \pi$, steady motion consistent with the boundary conditions at infinity cannot occur. For $\beta = n\pi$, $(n = 1, 2, 3, ...)$ solutions do not exist at all. It may be just be mentioned that when $\beta > \pi$ and $\neq \pi$, solutions with wavy terms exists, which do in fact, violate the boundary conditions at infinity. Here in the infinite channel flow, the value of $F = \dfrac{1}{\pi}$ (i.e. $\beta = \pi$) is found to be the critical value of the internal Froude number. Near the sink, the density stratification has no effect on the flow.

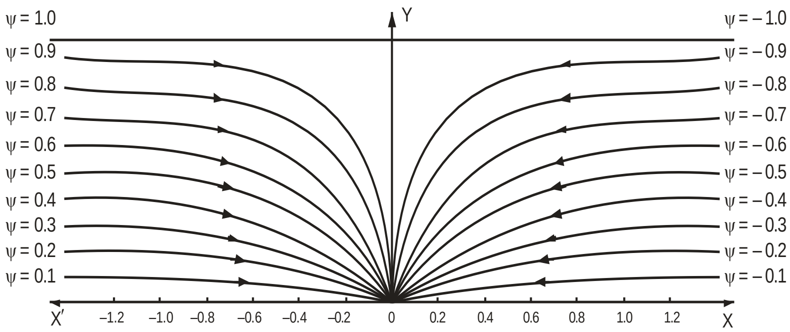

Fig. 1 Streamlines for the flow towards a sink placed at the origin for =ρ0.0

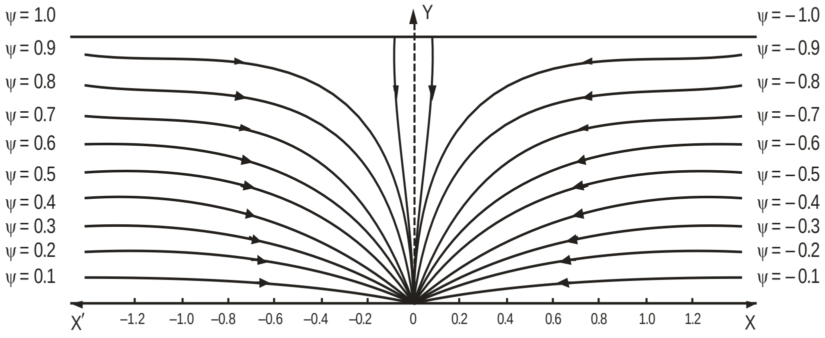

Fig. 2 Streamlines for the flow towards a sink placed at the origin for =ρ1.0

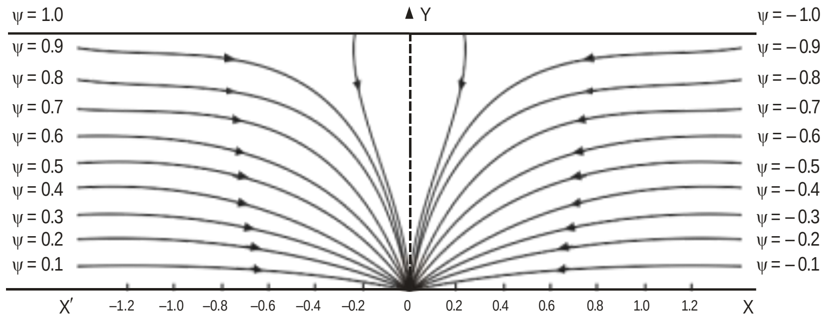

Fig. 3 Streamlines for the flow towards a sink placed at the origin for =ρ2.0

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