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Innovative Perspectives On Edge Domination Decomposition In Graphs

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ABSTRACT

A decomposition $\{G_1, G_2, \ldots, G_n\}$ is said to be an edge domination decomposition of a connected graph G if $\gamma'(G_j) = j, 1 \leq j \leq n$ where $\gamma'(G)$ is the edge domination number of G. In this study, we provide some findings about graphs edge domination decomposition.

Keywords: *Decomposition, Edge Domination, Edge Dominating Set, Edge Domination Number*

1. INTRODUCTION

Only simple, finite, and undirected graphs are taken into consideration. Let G be a graph with edge set $E(G)$and vertex set $V(G)$and let $p = |V(G)|$ and $q = |E(G)|$. For standard notations and terminologies, we refer [5]. Mitchell and Hedetniemi [4] proposed the idea of edge domination. Merly and Jothi [2] introduced Connected Domination Decomposition of Graphs. Motivated by the above we have introduced the new concept Edge Domination Decomposition of Graphs. The definitions and theorems which are needed in the article are given below:

Definition 1.1. [6]If every edge not in F is adjacent to at least one edge in F, then a subset F of E is referred to be an edge dominating set of G. The minimal cardinality taken over all edge dominating sets of G is the edge domination number $\gamma'(G)$ of G.

Definition 1.2. [1]If there are no two adjacent edges in set F, then an edge dominating set $F$is referred to as an independent edge dominating set.The minimal cardinality taken over all independent edge dominating sets of G is the independent edge domination number $\gamma_i'(G)$ of G.

Definition 1.3. [1]The number of edges in a maximal independent set of edges of G is a representation of the edge independence number, or $\beta_1(G)$.

Definition 1.4. [6]An edge's uv degree is defined as $degu + degv - 2$. An edge is called an isolated edge if $deg(uv) = 0$. An edge $uv \in E(G)$ is isolated if $deg(u) = deg(v) = 1$. The edge in the complete graph $K_2 = P_2$ is an isolated edge. $\Delta'(G)$ denotes the maximum degree among the edges of G.

Definition 1.5. [6] The open edge neighbourhood $N(e)$ of any edge $e \in E(G)$ is the set of edges adjacent to e. The closed neighbourhood $N[e]$ of an edge $e \in E(G)$is$N(e) \cup \{e\}$.

Definition 1.6. [2]A graph's decomposition is a set of edge-disjoint subgraphs $\{G_1, G_2, \ldots, G_n\}$ such that each edge in G belongs to exactly one of the subgraphs G_j, $1 \leq j \leq n$.

Theorem 1.7. [3] For any graph G, $\gamma'(G) \leq q - \Delta'(G)$.

Theorem 1.8. [3] Let F be an edge dominating set of G such that $|F| = \gamma'(G)$ then $|E(G) - F| \leq \sum_{e \in F} d(e)$.

Theorem 1.9. [3] If G is a (p, q) graph without isolated vertices, then $\frac{q}{\Delta'(G)+1} \leq \gamma'(G)$.

Theorem 1.10. [3] For any graph G, $\gamma'(G) \leq q - \beta_1 + q_0$, where q_0 is the number of isolated edges of G.

Theorem 1.11. [1] For any graph G, $\gamma'(G) = \gamma_i'(G)$.

2. EDGE DOMINATION DECOMPOSITION OF GRAPHS

Definition 2.1. For a connected graph G, an edge domination decomposition (EDD) is defined as a decomposition $\{G_1, G_2, \ldots, G_n\}$ if $\gamma'(G_j) = j, 1 \leq j \leq n$. Obviously $\sum_{j=1}^{n} \gamma'(G_j) = \frac{n(n+1)}{2}$.

Definition 2.2. A graph G is referred to be an edge domination decomposable graph (G_{EDD}) if it admits EDD $\{G_1, G_2, \ldots, G_n\}$.

Example 2.3. A graph G and its edge domination decomposition $\{G_1, G_2\}$ is depicted in the figure1:

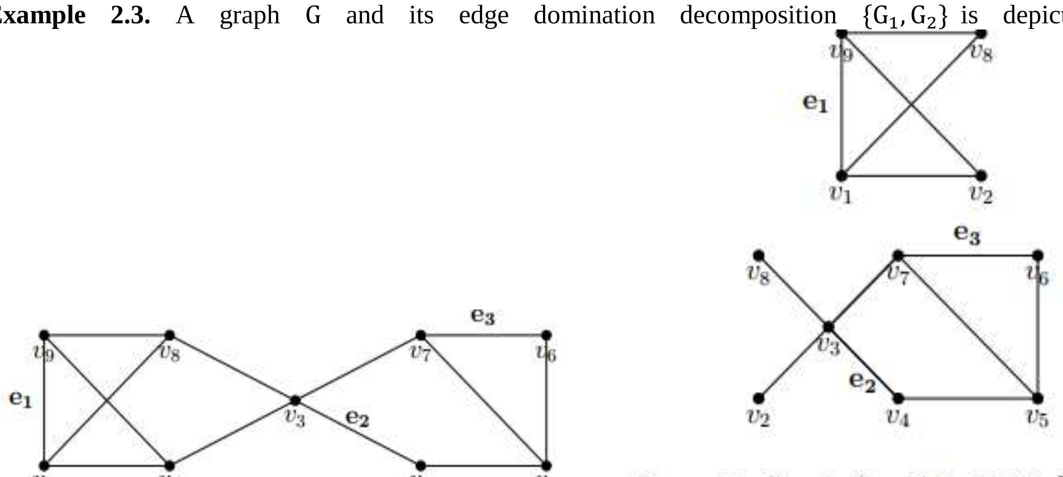

Figure 1: Graph G and its EDD $\{G_1, G_2\}$

Theorem 2.4.G admits EDD $\{G_1, G_2, \ldots, G_n\}$ if and only if $\gamma'(G) = \sum_{j=1}^{n} \gamma'(G_j)$.

Proof. Assume that G admits EDD $\{G_1, G_2, \ldots, G_n\}$. We claim that $\gamma'(G) = \sum_{j=1}^{n} \gamma'(G_j)$. We have$\gamma'(G_j) = j, 1 \leq j \leq n$. $\gamma'(G_1) + \gamma'(G_2) + \cdots + \gamma'(G_n) = 1 + 2 + \cdots + n$. $\sum_{j=1}^{n} \gamma'(G_j) = \frac{n(n+1)}{2}$.Let $F = \{e_1, e_2, \ldots, e_t\}$ be the edge dominating set of G where $t = \frac{n(n+1)}{2}$. Clearly each G_j has j edges in F. Therefore |F|=1+2+…+n. That is $\gamma'(G) = \frac{n(n+1)}{2}$. Hence $\gamma'(G) = \sum_{j=1}^{n} \gamma'(G_j)$.Conversely, assume $\gamma'(G) = \sum_{j=1}^{n} \gamma'(G_j)$. Since $\gamma'(G_j) = j$ for each j, $G_j$exists. Hence G admits EDD $\{G_1, G_2, \ldots, G_n\}$.

Remark 2.5. The graph G admits EDD $\{G_1, G_2, \ldots, G_n\}$ if and only if$n = \frac{-1+\sqrt{1+8\gamma'(G)}}{2}$.

Remark 2.6. The graph G admitsEDD$\{G_1, G_2, \ldots, G_n\}$ if and only if $\gamma'(G) = \frac{(2n+1)^2-1}{8}$.

Result 2.7. For any graph G_{EDD},$\sum_{j=1}^{n} \Delta'(G_j) \leq q - \gamma'(G)$.

By theorem 1.7 [3], For any graph G, $\gamma'(G) \leq q - \Delta'(G)$ we have $\gamma'(G_1) \leq q_1 - \Delta'(G_1) \ldots \gamma'(G_n) \leq q_n - \Delta'(G_n)$ adding we get $\sum_{j=1}^{n} \Delta'(G_j) \leq q - \gamma'(G)$.

Theorem 2.8. IfF is an edge dominating set ofG_{EDD},then (i)$F = \bigcup_{j=1}^{n} F_j(ii)\sum_{j=1}^{n} |E(G_j) - F_j| \leq \sum_{e \in F} d(e)$where F_j is the edge dominating set of G_j.

Proof. (i) Suppose $F \neq \bigcup_{j=1}^{n} F_j$ then $\gamma'(G) \neq \sum \gamma'(G_j)$,$\gamma'(G) \neq \frac{n(n+1)}{2}$ which is a contradiction. Hence $F = \bigcup_{j=1}^{n} F_j$. (ii) By theorem 1.8 [3], $|E(G) - F| \leq \sum_{e \in F} d(e)$.Since F_j is the edge dominating set of G_j for $j = 1, 2, 3 \ldots n$ we have $|E(G_j) - F_j| \leq \sum_{e \in F_j} d(e)$ for $j = 1,2,3 \ldots n$.Adding, $\sum_{j=1}^{n} |E(G_j) - F_j| \leq \sum_{j=1}^{n} \sum_{e \in F_j} d(e)$. Also since $F = \bigcup_{j=1}^{n} F_j$. we get $\sum_{j=1}^{n} |E(G_j) - F_j| \leq \sum_{e \in F_j} d(e)$.

Result 2.9. For a graph G_{EDD},$\sum_{j=1}^{n} \gamma'(G_j)\Delta'(G_j) \geq q - \gamma'(G)$

By [1.9] $\frac{q}{\Delta'(G)+1} \leq \gamma'(G)$. Therefore $\frac{q_j}{\Delta'(G_j)+1} \leq \gamma'(G_j)$

Hence $\sum \gamma'(G_j)\Delta'(G_j) \geq q - \gamma'(G)$.

Theorem 2.10. Let G be a graph with EDD$\{G_1, G_2, \ldots, G_n\}$ then

(i) $\beta_1(G) \leq \gamma'(G)\Delta'(G) + 1$.

(ii) $\sum_{j=1}^{n} \beta_1(G_j) \leq \sum_{j=1}^{n} \gamma'(G_j)\Delta'(G_j)$

Proof. Case(i) If q(G)=1 then $q_0(G_1) = 1$. Hence $\beta_1(G) \le \gamma'(G)\Delta'(G) + 1$.

Case (ii) q > 1 By theorem 1.10 [3], $\gamma'(G) \le q - \beta_1 + q_0$ where q_0 is the number of isolated edges of G. Since G is connected, $q_0 = 0$. So we have $\gamma'(G_1) \le q_1 - \beta_1(G_1)$,

$\gamma'(G_2) \le q_2 - \beta_1(G_2), \ldots.\gamma'(G_n) \le q_n - \beta_1(G_n)$, q_j be the size of G_j, $1 \le j \le n$.

Thus, $\gamma'(G) \le q - \sum_{j=1}^{n} \beta_1(G_j) \ldots (1)$. Since $\frac{q}{\Delta'(G)+1} \le \gamma'(G)$[1.10],

$q \le \sum_{j=1}^{n} \gamma'(G_j)\Delta'(G_j) + \gamma'(G) \ldots (2)$. From (1) and (2),

$\sum_{j=1}^{n} \beta_1(G_j) \le \sum_{j=1}^{n} \gamma'(G_j)\Delta'(G_j)$.

Theorem 2.11. If G admitsEDD$\{G_1, G_2, \ldots, G_n\}$ then $\gamma'(G_j) = \gamma_i'(G_j)$.

Proof. By theorem 1.11 [1], For any graph G, $\gamma'(G) = \gamma_i'(G)$. Therefore,

$\gamma'(G_j) = \gamma_i'(G_j)$.

3. CONCLUSION

In this study, we have introduced the new concept edge domination decomposition of graphs. Here we obtained some theorems and results for edge domination decomposition together with the characterization of graphs. Further this concept can be expanded to determine the bounds in graph parameters.

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