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$I\xi$ irresolute functions ideal topological spaces

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ABSTRACT

In this paper, we introduce and study a new class of function called $I\xi$ irresolute function in Ideal topological spaces.

Keywords: *Ideal topological space, I_ξ open set, I_ξ continues function and I_ξ irresolute function.*

1. INTRODUCTION

In point set topology continuous properties are the essential part. In the year 1982 Hdeib [3] introduced the new type of open set namely ω-open set and ω-closed function and after seven years in 1989 Hdeib [3] studied ω-continuous function. A subset A belonging to an ideal topological space $(X,\tau,I)$is said to be ω-open if for every $x\in A$, there exists an open set U_x containing x such that $U_x - A$ is countable. A function $f:(X,\tau)\to(Y,\sigma)$ where (X,τ) and (Y,σ) are topological spaces is ω- continuous if for every $x\in X$ and for every open set G in Y containing $f(x)$, there exists ω-open set U in X such that $f(U)\subseteq G$. In the year 2009 Noiri and Noorani [7] gave the pre-ω-open using ω-open. A subset A belonging to the topological space is said to be pre-ω-open if $A\subseteq int(cl(A))$ where $Int_\omega(A)$ is ω-interior operator of A in the space X. A function f : $(X,\tau)\to(Y,\sigma)$is pre-ω-continuous if $f^{-1}(U)$ is pre- ω-open set in (X,τ,I) for every open set U in Y.

Term ideal in topological space was first coined by Kuratowski and Vaidyanathaswamy [5,10]. Jankovic and Hamlett [3] were given the concept of I open sets in ideal topological spaces. An ideal I on a non empty collection of subsets of X which satisfies the following axioms:

(i) $S_1\in I$ and $S_2\subseteq S_1\Rightarrow S_2\in I$,

(ii) $S_1\in I$ and $S_2\in I\Rightarrow S_1\cup S_2\in I$ [5,8]. Applications of ideal into various fields examined by Jankovic and Hamlett [4]. A topological space (X,) together with an ideal Ion X and if $P(X)$ is the power set of X, a set operator $(.)^*:P(X)\to P(X)$, is said to be a local function [5] of S with respect to an τ and ideal I is defined as follows:

For $S\subseteq X$, $S^*(I,\tau)=\{s\in X|U\cap S\notin I$ for every $U\in\tau(s)$, where$\tau(s)=\{U\in\tau\,|s\in U\}$.

Further more $cl^*(S)=S\cup S^*(I,\tau)$defines a Kuratowski [5] closure operator for the topology

$\tau^*$is finer than τ.

Definition 1.1 [8]

Let(X,τ,I)be an ideal topological space and $S\subseteq X$.

(i) The I_ξ closure operator of is denoted by $I_\xi c(S)$ and it is defined as the intersection of all I_ξ closed sets containing S. That is, l $(S)=^{\xi}\cap\{P\subseteq X|S\subseteq P$ and $S\in\tau^{int}*C\}$.

(ii) The I_ξ interior operator of S is denoted by $I_\xi(S)$ and it is defined as the union of all

I_ξ open sets contained in S. That is, $int\,(S)=\cup\{P\subseteq X|P\subseteq S$ and $S\in\tau^{int}*\}$.

Definition 1.2 [9] ξ

Let (X,τ,I) be an ideal topological space and (Y,σ)be a topological space. Then the function f :$(X,\tau,I)\to(Y,\sigma)$ is said to be $I_{\xi\xi}$continuous if for each open set T in Y, $f^{-1}(T)$ is $I_\xi$open set in X.

Remark 1.3 [9]

Every open set is $I_\xi$open set, so every continuous function is $I_\xi$continuous.

2. $I\xi$ IRRESOLUTE FUNCTION IN IDEAL TOPOLOGICAL SPACE

In this section, we present and study I_ξ irresolute function and also investigate its characteristics.

Definition 2.1

Let X and Y be two ideal topological spaces. Then the function f : X $\to Y$ is said to be I_ξ irresolute if for each J_ξ open set T in Y, $f^{-1}(T)$ is I_ξ open set in X.

Example 2.2

Let(X,τ,I) and (Y,σ,J) be two ideal topological spaces, X=Y={s_1,s_2,s_3}, τ=σ={$\emptyset$,{s_1},{s_2},{s_1,s_2},Y}, with ideals I={$\emptyset$,{s_1}} and J={$\emptyset$,{s_2}}. Then $\tau^{int}*$={$\emptyset$,{s},{s},{s,s},{s,s},Y} and $\sigma^{int}*$={$\emptyset$,{s},{s},{s,s},{s,s},Y}.

We define an identity function f from (X,τ,I) into (Y,σ,J) by $f(s_1)$=s_1, $f(s_2)$=s_2 and $f(s_3)$ = s_3 and inverse function f is defined by $f^{-1}(s_1)$=s_1, $f^{-1}(s_2)$=s_2 and $f^{-1}(s_3)$= s_3. Here inverse image each $J_{\xi\xi}$ open set is $I_{\xi\xi}$ open set in X. Hence f is $I_{\xi\xi}$ irresolute function.

Theorem 2.3

Let a function f:(X,τ,I)$\to$(Y,σ,J) be $I_{\xi\xi}$irresolute if and only if $f^{-1}(S)$is $I_{\xi\xi}$ closed in X for each $J_{\xi\xi}$ closed set in Y.

Proof.

Let f be $I_{\xi\xi}$ irresolute function. If T is $J_{\xi\xi}$ closed set in Y,then$f^{-1}(T)$is $I_{\xi\xi}$ closed set in

X. By definition 2.1, $f^{-1}(Y- T)$ is $I_{\xi\xi}$ open set in X which implies $Y- f^{-1}(T)$ is $J_{\xi\xi}$ open in Y, therefore $f^{-1}(T)$ is $J_{\xi\xi}$ closed in Y.

Conversely, suppose$f^{-1}(T)$is $I_{\xi\xi}$ closed in X for each $J_{\xi\xi}$ closed set in Y. Let S be $I_{\xi\xi}$ open in X. Then X –S is $I_{\xi\xi}$ closed in X. By our assumption, f^{-1}(X –S) is $I_{\xi\xi}$ closed in X this implies that X –$f^{-1}(S)$ is $I_{\xi\xi}$ closed in X, therefore $f^{-1}(S)$ is $I_{\xi\xi}$ open in X. Hence f is $I_{\xi\xi}$ irresolute function.

Theorem 2.4

Every $I_{\xi\xi}$ irresolute function is $I_{\xi\xi}$ continuous function.

Proof.

Let$f$:(X,τ,I)$\to$(Y,σ,J) be $I_{\xi\xi}$ irresolute function. If T is $J_{\xi\xi}$ open in Y, then $f^{-1}(T)$ is $I_{\xi\xi}$ open in X. By definition f is $I_{\xi\xi}$ continuous function.

In the above theorem, if we take domain and co domain are both (X,τ,I) and (Y,σ,J)or same then we get, every $I_{\xi\xi}$ irresolute function is $I_{\xi\xi}$ continuous function or if we take (X,τ,I) and (Y,σ) are domain and co domain, in this case we get only f is $I_{\xi\xi}$ continuous function and we will never get f is $I_{\xi\xi}$ irresolute function.

Theorem 2.7

Let$f$:(X,τ,I)$\to$(Y,σ,J) and g:(Y,σ,J)$\to$(Z,ρ,K) be two $I_{\xi\xi}$ irresolute and $J_{\xi\xi}$ irresolute functions. Then gof is a $I_{\xi\xi}$ irresolute function if for each $K_{\xi\xi}$ open set in Z.

Proof.

Let S be $K_{\xi\xi}$ open in Z. Since f and g are $I_{\xi\xi}$ irresolute and $J_{\xi\xi}$ irresolute functions, by definition 2.1, $f^{-1}(S)$ is $J_{\xi\xi}$ open in Y and so $f^{-1}(g^{-1}(S)$) is $I_{\xi\xi}$ open in X. That is, $(g\circ f)^{-1}(S)$ is $I_{\xi\xi}$ open in X which gives gof is $I_{\xi\xi}$ irresolute function.

Theorem 2.8

If a function f :(X,τ,I)$\to$(Y,σ,J) is $I_{\xi\xi}$ irresolute if and only if for each $s\in$X and each $J_{\xi\xi}$ open set V in Y with$f(s)\in V$,there exists $I_{\xi\xi}$ open set U in (X,τ,I) such that $s\in U$ and $(U)\subseteq V$.

Proof.

Assume that f is $I_{\xi\xi}$ irresolute function. Let $s \in$ X and V be any $J_{\xi\xi}$ open set in Y containing (s). Put U=$f^{-1}(V)$. Since f is $I_{\xi\xi}$ irresolute function, then U is $I_{\xi\xi}$ open set in (X,τ,I) such that $s\in U$ and $f(U)\subseteq V$.

Conversely, Let V be any $I_{\xi\xi}$ open set in Y. For each $s\in f^{-1}(V)$, $f(s)\in V$. Then by hypothesis, there exists $J_{\xi\xi}$ open set U_x in (X,τ,I) such that $s \in U_x$ and $f(U) \subseteq V$. This implies that $U_x\in f^{-1}(V)$ and so $f^{-1}(V)$=$\cup x \in f-1(V)U_x$. Hence by theorem 3.11[1], $f^{-1}(V)$ = $\cup x \in f-1(V)U_x$ is $I_{\xi\xi}$ open set in(X,τ,I). Therefore, $f$is $I_{\xi\xi}$irresolute function.

Definition 2.9

A function f :(X,τ,I)$\to$(Y,σ,J) is said to be $I_{\xi\xi^*}$ open if the image of each $I_{\xi\xi}$ open set in X is a $J_{\xi\xi}$ open set in Y. A function f :(X,τ,I)$\to$(Y,σ,J)is said to be $I_{\xi\xi^*}$ closed if the image of each $I_{\xi\xi}$ closed set in X is a $J_{\xi\xi}$ closed set in Y.

Example 2.10

In example2.1

(1) The $I_{\xi\xi}$ open sets $\{s_1\}$, $\{s_2\}$, $\{s_1,s_2\}$ and $\{s_1,s_3\}$ are having $J_{\xi\xi}$ open images $\{s_3\}$, $\{s_2\}$, $\{s_{1,3}\}$and$\{s_2,s_3\}$.

(2) The $I_{\xi\xi}$ closed sets $\{s_2,s_3\}$,$\{s_1,s_3\}$,$\{s_3\}$ and $\{s_2\}$ are having $J_{\xi\xi}$ closed images $\{s_1,s_2\}$, $\{s_{1,3}\}$,$\{s_1\}$ and $\{s_2\}$.

Theorem 2.11

If a one - one onto$I_{\xi\xi}$irresolute function f :$(X,\tau,I) \to (Y,\sigma,J)$, then the following are equivalent:

1. $f^{-1}:(Y,\sigma,J)\to(X,\tau,I)$ is $I_{\xi\xi}$ irresolute,
2. f is $I_{\xi\xi^*}$ open,
3. f is $I_{\xi\xi^*}$ closed.

Proof.

(1)⇒(2).

Let f^{-1} be $I_{\xi\xi}$ irresolute function. Let S be $I_{\xi\xi}$ open in X. By definition 2.1, $(f^{-1})^{-1}(S)=f(S)$ is $J_{\xi\xi}$ open in Y. By definition 2.9, f is $I_{\xi\xi^*}$ open.

(2)⇒(3).

Assume $f$is $I_{\xi\xi}$ open. If T is $J_{\xi\xi}$ closed in Y, then $Y-T$ is $J_{\xi\xi}$ open in Y. By definition 2.9, $(Y-T)=Y-f(T)$is$J_{\xi\xi}$ open in $Y$and so $f(T)$ is $J_{\xi\xi}$ closed in Y. By definition 2.9, f is $I_{\xi\xi^*}$ closed.

(3)⇒(1).

Let f be $I_{\xi\xi}$ closed. If S is $I_{\xi\xi}$ closed in X, then by definition 2.9, $f(S)$ is $J_{\xi\xi}$ closed in Y and so $(f^{-1})^{-1}(S)$ is $J_{\xi\xi}$ closed in Y.By theorem 2.3, f^{-1} is $I_{\xi\xi}$ irresolute function.

Theorem 2.12

Let $f:(X,\tau,I) \to (Y,\sigma,J)$ and $g:(Y,\sigma,J)\to(Z,\rho,K)$ be two functions such that $g\ o\ f : (X,\tau,I) \to (Z,\rho,K)$ is $I_{\xi\xi^*}$ open. Then, if

1. f is $I_{\xi\xi}$ irresolute function and onto, then g is $I_{\xi\xi^*}$ open.
2. g is $J_{\xi\xi}$ irresolute and one-one, then f is $I_{\xi\xi^*}$ open.

Proof.

(1) ⇒(2).

Let T be $J_{\xi\xi}$ open in Y. Since f is $I_{\xi\xi}$ irresolute, by definition 2.1, $f^{-1}(T)$ is $I_{\xi\xi}$ closed in

X. Since gof is $I_{\xi\xi^*}$ open, by definition 2.10, $(gof)f^{-1}(T)$ is $K_{\xi\xi}$ open in Z. Since f is onto, $(gof)f^{-1}((T))=g(f(f^{-1}(T))=g(T)$ which gives $g(T)$ is $K_{\xi\xi}$ open in Z.Therefore g is $I_{\xi\xi^*}$ open.

(2) ⇒(3).

Let S be $I_{\xi\xi}$ open in X. Since $(gof)(S)$ is $K_{\xi\xi}$ open in Z, by definition 2.10. Again, since g is $J_{\xi\xi}$ irresolute and one-one, by definition 2.1,$g^{-1}(g(f(S))) = f(S)$ is $J_{\xi\xi}$ open in Y. Therefore f is $I_{\xi\xi^*}$ open.

Theorem 2.13

Let $f:(X,\tau,I)\to(Y,\sigma,J)$ and $g:(Y,\sigma,J)\to(Z,\rho,K)$ be two functions such that $gof:(X,\tau,I) \to (Z,\rho,K)$ is $I_{\xi\xi}$ closed. Then, if

1. f is $I_{\xi\xi}$ irresolute function and onto, then g is $I_{\xi\xi^*}$ closed.
2. g is $J_{\xi\xi}$ irresolute and one-one, then f is $I_{\xi\xi^*}$ closed.

Proof.

The proof is similar to that of the theorem 2.12 by changing $I_{\xi\xi}$ open to $I_{\xi\xi}$ closed.

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