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Edge Signal Domination In Some Transformation Graphs

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ABSTRACT

The concept of domination is an important part of graph theory. The edge signal domination is a parameter that takes into account the degree of vertices as well as the edge coverage in graphs which are crucial in the field of networking. Transformation graph is a concept in mathematics that primarily focuses on the structural change and behavior by applying graph-theoretic transformations to the graphs. It is mainly used in fields like computational modelling, biological model systems and other fields. In this article, we rely on the edge signal domination parameter and identify the parametric values for three distinct operations on graphs namely line graph, middle graph and corona product in some common graph families.

Keywords: *Edge Signal domination number, Line graph, Middle graph and Corona product graph.*

1. INTRODUCTION

The concept of line graph was introduced by the mathematician Abraham Albert in the year 1911 in a paper titled "A Graph-theoretical Definition of the Line Graph". He realised that a graph could be transformed into a new graph by considering the edges of the original graph as vertices in a new graph with adjacency defined by shared vertices of edges in the original graph. It was widely studied in the 1970s and onwards as a result of increasing interest in graph algorithms and network theory. Researchers began to explore how line graphs could be used in practical applications like network design, scheduling, graph coloring, computer networks, parallel processing, and database theory. During the late 20th and early 21st centuries, line graphs found applications in bioinformatics, social network analysis and graph-based machine learning.

The concept of the middle graph was formally introduced in the 1970s. In particular, the middle graph operation was recognized as a graph transformation that transforms the structure of a graph by subdividing edges. Researchers began to investigate how this transformation affects graph properties such as planarity, graph coloring and connectivity. The process of subdividing edges proved to have significant effects on graph properties often simplifying or complicating certain aspects like connectivity or coloring.

In 1967, while investigating novel operations on graphs, L. Roussopoulos proposed the Corona product in graph theory. His objective was to investigate methods for creating new graphs from pre-existing ones with an emphasis on creating graphs with special qualities. The Corona product was designed to create more intricate structures that could be examined for their combinatorial characteristics in addition to just combining graphs. The Corona product is an interesting and useful graph operation where two graphs are combined in a way that enhances their structure. It is applied in designing communication networks and parallel computing architectures, where the structure of a graph can represent the connectivity of nodes or processors in a system.

The concept of distance plays a major role in graph theory. The distance between any two vertices u and v in a graph G is the minimum number of edges in a path from u to v. The shortest possible path between any two vertices in a graph is known as geodesic and the length of the shortest path is called as the geodetic distance. This is essentially the minimal path in terms of the number of edges. A detour is a path between two vertices that is longer than the geodesic and its length is considered as detour distance refer to [5]. Later in the year 2010, K.M. Kathiresan [10] introduced signal distance which uses the degree of vertices in a graph along with the geodesic to find the shortest route. Further research on signal distance evolved it into signal number and edge signal number refer to [2] and [3] respectively. For further study regarding the distances, refer to [4].

Following the lead, Jachin Samuel. S and S. Angelin Kavitha Raj studied the signal number and introduced new combination of domination and signal number known as the signal domination number refer to [7]. This was because domination theory plays a major role in every day-to-day activity. In a graph G, a subset D of V is said to be a dominating set of G if every vertex in $V - D$ is adjacent to at least one vertex in D and the minimum cardinality of D is called as the domination number of G. For a detailed study on dominating sets, refer to [13]. Later, edge signal domination number came into existence due to its coverage of the entire graph refer to [9]. In this paper, we study the properties of edge signal domination number and implement them to find out their values in certain transformation graphs like line graph, middle graph and corona product graph. For graph terminologies, refer [6]. Throughout the paper, we consider G to be a connected graph.

2. PRELIMINARIES

Definition 2.1. [12] The signal distance $d_{SD}(u, v)$ between the vertices u and v in a graph G is defined as $d_{SD}(u, v) = \min_{S}\{d(u, v) + (\deg(u) - 1) + (\deg(v) - 1) + \sum_{w \in u-v}(\deg(w) - 2)\}$, where S is a path between the pair of vertices u and v, $d(u, v)$ is the length of the path S and w indicates the internal vertices of S. The signal path between u and v is called as the geosig path.

Definition 2.2. [2] The subset $S(G) \subseteq V(G)$ is called the signal set if every vertex in G lies in a geosig path between the vertices in S and its smallest cardinality is known as signal number. It's annotation is $sn(G)$.

Definition 2.3. [8] The subset $S \subseteq V$ is called the edge signal set if every edge lies in some geosig path between the vertices in S and S with least elements is called the edge signal number of a graph. It is denoted by $sn_1(G)$.

Definition 2.4. [3] A subset of V is called a signal dominating set if it is a dominating set as well as a signal set. The smallest size of all the signal dominating sets is recognized as signal domination number and is denoted by $\gamma_{sn}(G)$.

Definition 2.5. [9] A subset $S \subseteq V$ is called an edge signal dominating set if S forms a dominating set and an edge signal basis set. The edge signal dominating set with the least size is called the edge signal domination number and it is denoted by $\gamma_{es}(G)$.

Definition 2.6. [14] Let G be a loop-less graph. We construct a graph $L(G)$ in the following way: The vertex set of $L(G)$ is in one to one correspondence with the edge set of G and two vertices of $L(G)$ are joined by an edge if and only if the corresponding edges of G are adjacent in G. The graph $L(G)$ is called the line graph or the edge graph of G.

Definition 2.7. [1] The middle graph $M(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices x, y in the vertex set of $M(G)$ are adjacent in $M(G)$ if one of the following conditions holds.

1. $x, y \in E(G)$ and x, y are adjacent in G.
2. $x \in V(G), y \in E(G)$ and x, y are incident in G.

Definition 2.8. [11] The corona product of two graphs G and H denoted by $G \circ H$ is a graph operation defined as the graph obtained by taking one copy of G and $|V(G)|$ copies of H and joining the i^{th} vertex of G to every vertex in i^{th} copy of H.

3. MAIN RESULTS

3.1 Line Graphs

In this section, we indulge in transforming some standard graphs into their corresponding edge graph using the line graph transformation and find out their edge signal domination number in a detailed way.

Theorem 3.1.1. The edge signal domination number of path graph P_n subjected to line graph transformation is $\gamma_{es}(L(P_n)) = \left\lceil \frac{n+1}{3} \right\rceil$.

Proof. Since the line graph transformation of P_n is P_{n-1}, the result is obvious.

Theorem 3.1.2. For any cycle C_n with $n \neq 3, 5$ the edge signal domination number after applying the middle graph transformation is $\gamma_{es}(L(C_n)) = \left\lceil \frac{n}{3} \right\rceil$.

Proof. Since $L(C_n) \cong C_n$, the proof is obvious.

Theorem 3.1.3. The edge signal domination number determined after undergoing the process of line graph transformation of star graph is given by $\gamma_{es}\left(L(K_{1,n})\right) = n$.

Proof. In accordance with the line graph transformation, we obtain a relation $L(K_{1,n}) \cong K_n$ and so $\gamma_{es}\left(L(K_{1,n})\right) = n$.

Theorem 3.1.4. For a bi-star graph $S_{p,q}$, the edge signal domination number of line graph transformation of $S_{p,q}$ is $\gamma_{es}\left(L(S_{p,q})\right) = p + q$.

Proof. Since the line graph transformation of a bi-star $S_{p,q}$ produces two complete graphs of order p and q whose vertices are connected to a single vertex, the proof is obvious.

Theorem 3.1.5. For any two integers m and n with $2 \leq m \leq n$, the line graph transformation of complete bipartite graph has $\gamma_{es}\left(L(K_{m,n})\right) = m(n-1)$ to be its edge signal domination number.

Proof. Following the line graph transformation, $K_{m,n}$ is transformed into $L(K_{m,n})$ which has m – identical, complete components K_n that are interlinked with one another by n number of edges. By selecting an arbitrary vertex from each component in such a way that the collected set of vertices forms an independent set, we obtain a minimum dominating set of $L(K_{m,n})$. It is obvious that the remaining vertices form a dominating set of $L(K_{m,n})$. So $mn - m$ number of vertices are found in $V\left(L(K_{m,n})\right) - \gamma(L(K_{m,n}))$. Furthermore, $m(n-1)$ number of vertices form an edge signal cover for $L(K_{m,n})$. and so $\gamma_{es}\left(L(K_{m,n})\right) \leq m(n-1)$.Suppose, $\gamma_{es}\left(L(K_{m,n})\right) < m(n-1)$, then there exists an edge in $L(K_{m,n})$ that is not covered by the geosig paths formed by the vertices of γ_{es} – set and so $\gamma_{es}\left(L(K_{m,n})\right) = m(n-1)$.

Theorem 3.1.6. The edge signal domination number of any complete graph after applying the line graph transformation is found to be $\gamma_{es}(L(K_n)) = \begin{cases} 3 & if\ \ n = 3 \\ 4(n-3) & Otherwise \end{cases}$.

Proof. For $n = 3$, the result follows from Theorem 3.1.2. Now we prove for $n > 3$. According to line graph definition, K_n is transformed into $L(K_n)$ which is regular with degree $2(n-2)$ and $|V(L(K_n)| = \frac{n(n-1)}{2}$. We prove this by induction on n. Let S be γ_{es} – set of $L(K_n)$. Take $n = 4$. Then we have the line graph transformation $L(K_4)$ having 6 vertices and 12 edges. Since $diam(L(K_n)) = 2$ for every $n \geq 4$, let $\{a, b\} \in S$ such that $d(a, b) = 2$. Then all the vertices in $L(K_4)$ is covered by $\{a, b\}$ and the edges incident with a and b are covered by the $a - b$ geosig path. The induced sub-graph of the non incident edges forms a 2 - regular graph with 4 vertices. So S contains $\{a, b, c, d\}$ where $d(c, d) > 1$ and hence the result is true for $n = 4$. Similarly for $n = 5$, we have $\{x, x_1, x_2, x_3\} \in S$ such that $d(x, x_i) = 2$ for $1 \leq i \leq 3$. Moreover, the induced sub-graph obtained from the non incident edges forms a 3 - regular graph with 6 vertices. By adding the vertices $\{y, y_1, y_2, y_3\}$ to S from the induced sub-graph, we obtain an edge signal dominating set. However, removing any one of the vertices from S causes S to lose its property of edge signal cover and therefore S is minimum edge signal dominating set. Assume that the result is true for some $n = k$. Now we prove for $n = k + 1$. Let $\{u, u_1, u_2, \ldots, u_{\frac{k^2-3k+2}{2}}\} \in S$ such that $d(u, u_i) = 2$ for $1 \leq i \leq \frac{k^2-3k+2}{2}$. It is obvious that S forms a γ_{sn} – set of $L(K_n)$ where, $n = k + 1$. Consider the induced sub-graph that is obtained by selecting the edges that are not incident with the vertices of S. Then the set $\{v, v_1, v_2, \ldots, v_{\frac{-k^2+11k-22}{2}}\} \in S$ from the induced sub-graph can form an edge signal basis set. Thus $\{v, v_1, v_2, \ldots, v_{\frac{-k^2+11k-22}{2}}, u, u_1, u_2, \ldots, u_{\frac{k^2-3k+2}{2}}\}$ forms a γ_{es} – set of $\gamma_{es}(L(K_n))$ where, $n = k + 1$.

3.2 Middle Graphs

In this section, we obtain some results regarding the edge signal domination number for some standard graphs that are transformed into their respective middle graphs.

Theorem 3.2.1. For every $n \geq 2$, the edge signal domination number of middle graph of path graph is given by $\gamma_{es}(M(P_n)) = n$.

Proof. Since $\gamma_{sn}(M(P_n)) = n$, we can deduce that $\gamma_{es}(M(P_n)) \geq n$. ..... (1)

Let $u_1, u_2, \ldots, u_n \in V(P_n)$ and $v_1, v_2, \ldots, v_{n-1} \in E(P_n)$. In accordance with the principles of middle graph transformation, we transform P_n into $M(P_n)$ whose vertex set is $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_{n-1}$. It is indisputable that the geosig path running between u_1 and u_n covers all the edges $v_i v_{i+1}$ $(1 \leq i \leq n-1)$. Further taking into consideration, the set of vertices $\{u_j \mid 2 \leq j \leq n-1\}$ is chosen so that the geosig path formed by any pair of these vertices cover the remaining edges of $M(P_n)$. So, $\{u_i \mid 1 \leq i \leq n\}$ forms an edge signal dominating set of $M(P_n)$. So, $\gamma_{es}(M(P_n)) \leq n$. ..... (2)

From equations (1) and (2), we attain the desired result.

Theorem 3.2.2. For any cycle C_n with $n \geq 3$, the edge signal domination number of middle graph associated with cycle graph is $\gamma_{es}\big(M(C_n)\big) = \begin{cases} n & if\ n > 3 \\ 5 & otherwise \end{cases}$.

Proof. For $n > 3$, according to the framework of middle graph transformation, $M(C_n)$ contains an outer cycle of length $2n$ and an inner cycle of length n formed by the edge set of C_n. So, it is easily identified that the vertices in $M(C_n)$ that corresponds to the vertices of C_n can form an edge signal dominating set in $M(C_n)$ and hence $\gamma_{es}\big(M(C_n)\big) \leq n$. Let S be γ_{es} – set of $M(C_n)$. Suppose, $\gamma_{es}\big(M(C_n)\big) < n$, then there is a vertex u_1 in S such that $S - \{u_1\}$ forms an edge signal dominating set in $M(C_n)$. Then it is obvious that u_1 lies in some geosig path formed by the vertices of $S - \{u_1\}$. However, no two u_i $(1 \leq i \leq n)$ is adjacent to each other in $M(C_n)$ which contradicts the principle of dominating set. Therefore $\gamma_{es}\big(M(C_n)\big) = n$. For $n = 3$, the structure of $M(C_n)$ provides an insight to identify the γ_{es} – set where $\Delta\big(M(C_n)\big) = 4$ and it is found to be 5.

Theorem 3.2.3. In the context of the middle graph transformation of star graph, the edge signal domination number is $\gamma_{es}\Big(M\big(K_{1,n}\big)\Big) = n + 1$.

Proof. Since every pendant vertices are contained in γ_{es} – set, $\gamma_{es}\Big(M\big(K_{1,n}\big)\Big) \geq n$. Since $M\big(K_{1,n}\big)$ contains a vertex x whose neighbourhood is adjacent to these pendant vertices, x lies in γ_{es} – set of $M\big(K_{1,n}\big)$. So $\gamma_{es}\Big(M\big(K_{1,n}\big)\Big) \geq n + 1$. It is evident that all the vertices are covered by γ_{es} – set. Furthermore, the entire edges incident with x and the pendant vertices are covered by the edge signal basis set. Since the geosig path formed by the pendant vertices do not cover the edges incident with x, we can deduce that all the edges are covered by γ_{es} – set of $M\big(K_{1,n}\big)$. So $\gamma_{es}\Big(M\big(K_{1,n}\big)\Big) = n + 1$.

Theorem3.2.4. The edge signal domination number of complete graph after being subjected to middle graph transformation is $\gamma_{es}\big(M(K_n)\big) = \begin{cases} \binom{n}{2} & if\ n > 3 \\ 5 & n = 3 \end{cases}$.

Proof. For $n = 3$, the result is obvious from Theorem 3.2.2. Let S be γ_{es} – set of $M(K_n)$. Now we assume $n > 3$. Since the transformation maps every edge in K_n to vertex in $M(K_n)$, it becomes clearer that $\gamma_{es}\big(M(K_n)\big) \leq \binom{n}{2}$. Suppose there exists a vertex x in S such that $S - \{x\}$ foms a γ_{es} – set of $M(K_n)$, then $\gamma_{es}\big(M(K_n)\big) < \binom{n}{2}$. It is evident that $\gamma_{es}\big(M(K_n)\big)$ cannot be less than n. If $x \in V(K_n)$, then $S - \{x\}$ does not form a signal cover for $M(K_n)$ and contradicts our assumption. So $x \in E(K_n)$ with $\deg(x) = 2(n-1)$ in $M(K_n)$. Clearly, x is adjacent to at least one of the vertices corresponding to $V(K_n)$ and so x is dominated. According to our assumption x is covered by the geosig path formed by the vertices of $S - \{x\}$. However, it is unclear that the edges xx_i for some $i = 1$ to $2(n-1)$ is covered by the edge signal basis of $S - \{x\}$. It is evident that the edges between x and the vertices corresponding to $V(K_n)$ are covered by the edge signal basis set say xx_1 and xx_2. Furthermore, $S - \{x\}$ contains some vertices that corresponds to $E(K_n)$ under the condition $d\big(x_i, u_j\big) = 2$ where u_j $(1 \leq j \leq n)$ are the vertices that corresponds to $V(K_n)$. This provides a contradiction to our assumption. Therefore $\gamma_{es}\big(M(K_n)\big) = \binom{n}{2}$.

3.3 Corona Product Graphs

This section involves the formation of corona product graphs from some common graph families and a comprehensive determination of their edge signal domination number.

Theorem 3.3.1. The edge signal domination number for the graph obtained by the corona product of cycle C_n and complete graph K_1 is $\gamma_{es}(C_n \odot K_1) = n$.

Proof. Since the edge signal dominating set contains all the pendant vertices, $\gamma_{es}(C_n \odot K_1) \geq n$. Clearly, all the vertices in $\gamma_{es}(C_n \odot K_1)$ is dominated and covered by the geosig paths formed by the pendant vertices. Moreover, the edges incident with the pendant vertices are also covered by the geosig paths of pendant vertices. It is enough to prove only if the edges in the cycle are covered by the geosig paths formed by the pendant vertices. Suppose there exist an edge that is not covered by any of the geosig paths, then there are two different geosig paths that covers the end points of that edge. This contradicts our assumption. So $\gamma_{es}(C_n \odot K_1) = n$.

Theorem 3.3.2. For the corona product of a complete graph K_n and K_1, the edge signal domination number is identified as $\gamma_{es}(K_n \odot K_1) = n$.

Proof. It is natural that all the vertices of $K_n \odot K_1$ are covered by the geosig paths formed by the pendant vertices. Also, the set of pendant vertices forms a dominating set of $K_n \odot K_1$. So $\gamma_{es}(K_n \odot K_1) \geq n$. Since every pair of vertices of K_n has a unique edge, all the edges are covered by the geosig paths of pendant vertices and so $\gamma_{es}(K_n \odot K_1) = n$.

Theorem 3.3.3. The edge signal domination number of corona product of path P_n and K_1 is given by $\gamma_{es}(P_n \odot K_1) = n$.

Proof. The proof is obvious.

Theorem 3.3.4. The edge signal domination number of the graph transformed from the cycle C_n and path P_m by the corona product operation is $\gamma_{es}(C_n \odot P_m) = nm$.

Proof. Let $\{u, u_1, u_2, \dots, u_n\}$ be the vertex set of C_n and $\{v_1, v_2, \dots, v_m\}$ be the vertex set of P_m. By the definition of corona product graph operation, we obtain n copies of P_m and let the vertex set be $\{v_{i1}, v_{i2}, \dots, v_{im}\}$ with $(1 \le i \le n)$. Let us consider the induced sub-graph of $C_n \odot P_m$ whose vertices are $u_1, v_{i1}, v_{i2}, \dots, v_{im}$ for some i. It is obvious that u_i acts as a universal vertex and so the edge signal domination number is m. This is true for every $(1 \le i \le n)$. So $\gamma_{es}(C_n \odot P_m) = nm$.

Theorem 3.3.5. The edge signal domination number of the graph obtained by the corona product operation from the cycle C_n and path P_m is given by $\gamma_{es}(P_m \odot C_n) = nm$.

Proof. The proof is similar to Theorem 3.3.4.

Theorem 3.3.6. The edge signal domination number of corona product of path K_n and P_m is given by $\gamma_{es}(K_n \odot P_m) = nm$.

Proof. The proof is identical to the Theorem 3.3.4.

Theorem 3.3.7. For the corona product of path P_m and complete graph K_n, the edge signal domination number is $\gamma_{es}(P_m \odot K_n) = nm$.

Proof. The result follows from Theorem 3.3.6.

Theorem 3.3.8. For any two connected graphs G and H, $\gamma_{es}(G \odot H)$ contains all the vertices of H.

Proof. Let G and H be any two connected graphs of order n_1 and n_2 respectively. By the definition of line graph transformation, we have n_1 copies of H each attached to a distinct vertex of G. Since each vertex in G is adjacent to every vertex of the corresponding cop of H, the edges between H and the particular vertex of G can only be covered by the geosig path formed by the vertices of H. So $\gamma_{es}(G \odot H)$ contains all the vertices of H.

Theorem. For any two connected graphs G_1 and G_2, $\gamma_{es}(G_1 \odot G_2) \ge max\{\gamma_{es}(G_1), \gamma_{es}(G_2)\}$.

Proof. Suppose $\gamma_{es}(G_1) \le \gamma_{es}(G_2)$, then by Theorem 3.3.8, the result is obvious. Suppose $\gamma_{es}(G_1) > \gamma_{es}(G_2)$. Then by Theorem 3.3.8, $\gamma_{es}(G_1 \odot G_2)$ contains all the vertices of G_2. Also if $\gamma_{es}(G_1) > |V(G_2)|$, then $\gamma_{es}(G_1 \odot G_2)$ contains all the vertices from each copy of G_2 which can cover all the edges between G_1 and G_2 as well as dominate every vertices of G_1. The geosig path formed by the vertices of G_2 with each ends taken from distinct copy can cover the edge set of G_1 which forms a minimum edge signal dominating set of $\gamma_{es}(G_1 \odot G_2)$.

4. CONCLUSION

By analyzing the line graph, middle graph, and corona product data together, we observe a few key trends and correlations. The line graph shows overall performance or trends over time which provides a broad view of how a particular variable or metric evolves. The middle graph, which compares different categories or regions, reveals how these variations influence the overall results. Corona product graph helps us understand the network-like structure of demand changes, the influence of specific time periods, and the interactions between various market factors. With these three main transformation graphs, it is a good opportunity to have a deeper insight on the edge signal domination number.

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