Journal of Neonatal Surgery
ISSN(Online): 2226-0439
Vol. 14, Issue 32s (2025)
https://www.jneonatalsurg.com

Analyzing Algebraic Operations And Properties Of Pentapartitioned Neutrosophic Binary Sets

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Cite this paper as: A. Anit Yoha, M. Jaslin Melbha, (2025) Analyzing Algebraic Operations And Properties Of Pentapartitioned Neutrosophic Binary Sets. *Journal of Neonatal Surgery*, 14 (32s), 4169-4176.

ABSTRACT

A Pentapartitioned Neutrosophic Binary Set is represented through a membership functions of truth, contradiction, ignorance, unknown, and falsity, which quantifies the degrees of membership functions for each element over two universes. This paper explores an algebraic operations defined on pentapartitioned neutrosophic binary set. The fundamental algebraic operations on pentapartitioned neutrosophic binary set namely algebraic sum, algebraic difference, algebraic product, algebraic quotient, scalar multiplication, and exponentiation. To further clarify, we include examples illustrating the implementation of the defined operations. Furthermore, these operations are also investigated for key properties, like commutativity and distributivity, and validated using mathematical proofs.

Keywords: *Pentapartitioned Neutrosophic Binary Set, Neutrosophic Set, Algebraic Sum, Algebraic Product, Exponentiation*.

1. INTRODUCTION

Neutrosophic sets, pioneered by Smarandache, enhance fuzzy sets by including truth, indeterminacy, and falsity degrees [19]. This advancement effectively resolves uncertainties, partial data in real-life contexts. But in some practical situations, neutrosophic membership grades are not enough to handle the full complexity of indeterminacies, pentapartitioned neutrosophic set is needed to distinguish between different types of indeterminacies within a single universe and it is proposed by Rama Mallick & Surapati Pramanik [16]. It contains five distinct membership functions by expanding the indeterminant function into three parts. This opens a way to model complex problems in a new perception where uncertainty involves different dimensions, and it offers a more detailed and accurate depiction of inconsistent information. Later, neutrosophic vague binary set is developed by Remya & Francina Shalini to solve problems that require solutions over two distinct universes [17]. Based on this, Surekha grounded the idea of neutrosophic binary set [22]. Recently, we pioneered the concept of pentapartitioned neutrosophic binary set assigned by five distinct membership functions over two universes to solve certain problems that require two universes instead of a single universe and to overcome the limitations of traditional neutrosophic sets, providing flexible framework for modeling complex uncertainty in both theoretical and practical contexts [1].

The single-valued neutrosophic set is pioneered by Wang et al., that simplifies computations and enhances algorithmic efficiency [24]. It aligns well with a data which is represented as single values in many real- life scenarios. Subsequently, Liu & Wang formulated various algebraic operations over single-valued neutrosophic sets, to further explore their mathematical structure and applications, ensuring consistency in applications like ranking, similarity measurement, and aggregation [13]. Also, Smarandache introduced new operations including subtraction and division in single valued neutrosophic numbers to extend the applicability of neutrosophic sets to a broader range of mathematical and practical problems, bridging the gap between neutrosophic theory and classical algebra [21]. Subtraction and Division are useful in scenarios like performance evaluation, scaling, error analysis and comparison of alternatives. Hazwani Hashim also formulated various basic operations in new interval neutrosophic vague sets, to contribute their mathematical development and practical use [10]. Further refining the concept, Ye proposed a simplified neutrosophic set, offering a more streamlined version of the traditional neutrosophic set and also he established subtraction and division operations over simplified neutrosophic sets [25,27]. This motivates us to propose these operations on pentapartitioned neutrosophic binary set namely algebraic sum, algebraic difference, algebraic product, algebraic quotient, scalar multiplication, and exponentiation over two different universes and provide examples to demonstrate the defined operations. Additionally, key properties of these algebraic operations have been rigorously proven, solidifying their theoretical robustness and practical relevance. The proposed framework enhances modeling capabilities for complex, real world problems involving interactions between distinct domains. It is particularly useful in scenarios where data or constraints originate from different universes, requiring a more sophisticated representation of uncertainty. This work advances neutrosophic theory and opens new research directions, such as extending the framework to multiple universes or integrating it with other uncertainty models.

2. PRELIMINARIES

This section provides an overview of fundamental definitions essential to the discussion.

Definition 2.1

A *neutrosophic set* $\tilde{\mathbb{N}}_{\mathcal{S}}$ defined in a universe $\tilde{\mathcal{S}}$ can be expressed as:

$$\tilde{\mathbb{N}}_{\mathcal{S}} = \{\langle \tilde{m}, \mu_{\tilde{A}}(\tilde{m}), \eta_{\tilde{A}}(\tilde{m}), \gamma_{\tilde{A}}(\tilde{m})\rangle : \tilde{m} \in \tilde{\mathcal{S}}\},$$

Here $\mu_{\tilde{A}}(\tilde{m}), \eta_{\tilde{A}}(\tilde{m}), \gamma_{\tilde{A}}(\tilde{m})$ correspond to the truth, indeterminant, and falsity membership degrees of each $\tilde{m} \in \tilde{\mathcal{S}}$. So, $0 \leq \mu_{\tilde{A}}(\tilde{m}) + \eta_{\tilde{A}}(\tilde{m}) + \gamma_{\tilde{A}}(\tilde{m}) \leq 3$.

Definition 2.2

Let $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$ represent two distinct universes. The *pentapartitioned neutrosophic binary set (PNBS)* $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})$ defined in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$ is expressed as follows:

$$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \left\{\begin{array}{c}\langle \tilde{m}, \mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m})\rangle, \\ \langle \tilde{n}, \mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n})\rangle : \tilde{m} \in \tilde{\mathcal{P}}, \tilde{n} \in \tilde{\mathcal{Q}}\end{array}\right\}$$

where the functions $\mu_{\tilde{A}_1}(\tilde{m})$, $\mu_{\tilde{A}_2}(\tilde{n})$ represent the truth-memberships; $\sigma_{\tilde{A}_1}(\tilde{m})$, $\sigma_{\tilde{A}_2}(\tilde{n})$ represent the contradiction-memberships; $\vartheta_{\tilde{A}_1}(\tilde{m})$, $\vartheta_{\tilde{A}_2}(\tilde{n})$ represent the ignorance-memberships; $\phi_{\tilde{A}_1}(\tilde{m})$, $\phi_{\tilde{A}_2}(\tilde{n})$ represent the unknown-memberships and $\gamma_{\tilde{A}_1}(\tilde{m})$, $\gamma_{\tilde{A}_2}(\tilde{n})$ represent the falsity-memberships for each $\tilde{m} \in \tilde{\mathcal{P}}$ and $\tilde{n} \in \tilde{\mathcal{Q}}$ such that

$0 \leq \mu_{\tilde{A}_1}(\tilde{m}) + \sigma_{\tilde{A}_1}(\tilde{m}) + \vartheta_{\tilde{A}_1}(\tilde{m}) + \phi_{\tilde{A}_1}(\tilde{m}) + \gamma_{\tilde{A}_1}(\tilde{m}) \leq 5$ and $0 \leq \mu_{\tilde{A}_2}(\tilde{n}) + \sigma_{\tilde{A}_2}(\tilde{n}) + \vartheta_{\tilde{A}_2}(\tilde{n}) + \phi_{\tilde{A}_2}(\tilde{n}) + \gamma_{\tilde{A}_2}(\tilde{n}) \leq 5$.

Definition 2.3

Let the *PNBS* $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \left\{\begin{array}{c}\langle \tilde{m}, \mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m})\rangle, \\ \langle \tilde{n}, \mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n})\rangle : \tilde{m} \in \tilde{\mathcal{P}}, \tilde{n} \in \tilde{\mathcal{Q}}\end{array}\right\}$

in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$, then their *complement* $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})^c$ is expressed as follows:

$$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})^c = \left\{\begin{array}{c}\langle \tilde{m}, \gamma_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), 1 - \vartheta_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \mu_{\tilde{A}_1}(\tilde{m})\rangle, \\ \langle \tilde{n}, \gamma_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), 1 - \vartheta_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \mu_{\tilde{A}_2}(\tilde{n})\rangle : \tilde{m} \in \tilde{\mathcal{P}}, \tilde{n} \in \tilde{\mathcal{Q}}\end{array}\right\}$$

Definition 2.4

Let the two *PNBS*s $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \left\{\begin{array}{c}\langle \tilde{m}, \mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m})\rangle, \\ \langle \tilde{n}, \mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n})\rangle : \tilde{m} \in \tilde{\mathcal{P}}, \tilde{n} \in \tilde{\mathcal{Q}}\end{array}\right\}$ and $(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \left\{\begin{array}{c}\langle \mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{B}_1}(\tilde{m})\rangle, \\ \langle \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{B}_2}(\tilde{n}), \vartheta_{\tilde{B}_2}(\tilde{n}), \phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{B}_2}(\tilde{n})\rangle : \tilde{m} \in \tilde{\mathcal{P}}, \tilde{n} \in \tilde{\mathcal{Q}}\end{array}\right\}$ in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \subseteq (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ iff $\mu_{\tilde{A}_1}(\tilde{m}) \leq \mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}) \leq \sigma_{\tilde{B}_1}(\tilde{m})$, $\vartheta_{\tilde{A}_1}(\tilde{m}) \geq \vartheta_{\tilde{B}_1}(\tilde{m})$, $\phi_{\tilde{A}_1}(\tilde{m}) \geq \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m}) \geq \gamma_{\tilde{B}_1}(\tilde{m})$ for every $\tilde{m} \in \tilde{\mathcal{P}}$ and $\mu_{\tilde{A}_2}(\tilde{n}) \leq \mu_{\tilde{B}_2}(\tilde{n})$, $\sigma_{\tilde{A}_2}(\tilde{n}) \leq \sigma_{\tilde{B}_2}(\tilde{n})$, $\vartheta_{\tilde{A}_2}(\tilde{n}) \geq \vartheta_{\tilde{B}_2}(\tilde{n})$, $\phi_{\tilde{A}_2}(\tilde{n}) \geq \phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n}) \geq \gamma_{\tilde{B}_2}(\tilde{n})$ for every $\tilde{n} \in \tilde{\mathcal{Q}}$.

Definition 2.5

Let the two PNBSs $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \begin{Bmatrix} \langle \tilde{m}, \mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m}) \rangle, \\ \langle \tilde{n}, \mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n}) \rangle \end{Bmatrix}$ and

$(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle \mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{B}_1}(\tilde{m}) \rangle, \\ \langle \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{B}_2}(\tilde{n}), \vartheta_{\tilde{B}_2}(\tilde{n}), \phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{B}_2}(\tilde{n}) \rangle \end{Bmatrix}$ in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then the union $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \cup (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ and intersection $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \cap (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ of two PNBSs are expressed as follows:

$$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \cup (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle \mu_{\tilde{A}_1}(\tilde{m}) \vee \mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}) \vee \sigma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}) \wedge \vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}) \wedge \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m}) \wedge \gamma_{\tilde{B}_1}(\tilde{m}) \rangle, \\ \langle \mu_{\tilde{A}_2}(\tilde{n}) \vee \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}) \vee \sigma_{\tilde{B}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}) \wedge \vartheta_{\tilde{B}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}) \wedge \phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n}) \wedge \gamma_{\tilde{B}_2}(\tilde{n}) \rangle \end{Bmatrix}$$

$$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \cap (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle \mu_{\tilde{A}_1}(\tilde{m}) \wedge \mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}) \wedge \sigma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}) \vee \vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}) \vee \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m}) \vee \gamma_{\tilde{B}_1}(\tilde{m}) \rangle, \\ \langle \mu_{\tilde{A}_2}(\tilde{n}) \wedge \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}) \wedge \sigma_{\tilde{B}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}) \vee \vartheta_{\tilde{B}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}) \vee \phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n}) \vee \gamma_{\tilde{B}_2}(\tilde{n}) \rangle \end{Bmatrix}$$

3. PENTAPARTITIONED NEUTROSOPHIC BINARY SET OPERATIONS

In this section, we present several fundamental operations defined on *PNBS*. The following are the definitions of operations over *PNBS* such as algebraic sum, algebraic difference, algebraic product, scalar multiplication, exponentiation, and algebraic quotient:

Definition 3.1

Let the two *PNBS*s $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \begin{Bmatrix} \langle \tilde{m}, \mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m}) \rangle, \\ \langle \tilde{n}, \mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n}) \rangle \end{Bmatrix}$ and

$(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle \mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{B}_1}(\tilde{m}) \rangle, \\ \langle \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{B}_2}(\tilde{n}), \vartheta_{\tilde{B}_2}(\tilde{n}), \phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{B}_2}(\tilde{n}) \rangle \end{Bmatrix}$ in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then the *algebraic sum* of $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})$ and $(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ denoted as $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \oplus (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ and is defined as follows:

$$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \oplus (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \left\langle \begin{matrix} \mu_{\tilde{A}_1}(\tilde{m}) + \mu_{\tilde{B}_1}(\tilde{m}) - \mu_{\tilde{A}_1}(\tilde{m})\mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}) + \sigma_{\tilde{B}_1}(\tilde{m}) - \sigma_{\tilde{A}_1}(\tilde{m})\sigma_{\tilde{B}_1}(\tilde{m}), \\ \vartheta_{\tilde{A}_1}(\tilde{m})\vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m})\phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m})\gamma_{\tilde{B}_1}(\tilde{m}) \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} \mu_{\tilde{A}_2}(\tilde{n}) + \mu_{\tilde{B}_2}(\tilde{n}) - \mu_{\tilde{A}_2}(\tilde{n})\mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}) + \sigma_{\tilde{B}_2}(\tilde{n}) - \sigma_{\tilde{A}_2}(\tilde{n})\sigma_{\tilde{B}_2}(\tilde{n}), \\ \vartheta_{\tilde{A}_2}(\tilde{n})\vartheta_{\tilde{B}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n})\phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n})\gamma_{\tilde{B}_2}(\tilde{n}) \end{matrix} \right\rangle \end{Bmatrix}$$

Example 3.2

Let $\tilde{\mathcal{P}} = \{\tilde{m}_1, \tilde{m}_2\}$ and $\tilde{\mathcal{Q}} = \{\tilde{n}_1, \tilde{n}_2\}$ be two universes and let

$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \begin{Bmatrix} \langle (\tilde{m}_1, .8, .59, .71, .63, .34), (\tilde{m}_2, .74, .11, .19, .25, .36) \rangle, \\ \langle (\tilde{n}_1, .72, .85, .92, .5, .22), (\tilde{n}_2, .11, .31, .42, .51, .16) \rangle \end{Bmatrix}$ and

$(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle (\tilde{m}_1, .79, .24, .5, .3, .1), (\tilde{m}_2, .5, .9, .61, .7, .81) \rangle, \\ \langle (\tilde{n}_1, .1, 0, .42, .25, .3), (\tilde{n}_2, .6, .83, .51, .3, .1) \rangle \end{Bmatrix}$ be two *PNBS*s over $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \oplus (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle (\tilde{m}_1, .958, .688, .355, .189, .034), (\tilde{m}_2, .87, .911, .115, .175, .291) \rangle, \\ \langle (\tilde{n}_1, .748, .85, .386, .125, .066), (\tilde{n}_2, .644, .882, .214, .153, .016) \rangle \end{Bmatrix}$

Definition 3.3

Let $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \begin{Bmatrix} \langle \tilde{m}, \mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m}) \rangle, \\ \langle \tilde{n}, \mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n}) \rangle \end{Bmatrix}$ and

$(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle \mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{B}_1}(\tilde{m}) \rangle, \\ \langle \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{B}_2}(\tilde{n}), \vartheta_{\tilde{B}_2}(\tilde{n}), \phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{B}_2}(\tilde{n}) \rangle \end{Bmatrix}$ be two *PNBS*s in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then, the *algebraic product* of $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})$ and $(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$denoted as $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \otimes (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ and is expressed as follows:

$$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \otimes (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) =$$

$$\left\{\begin{matrix} \left\langle \begin{matrix} \mu_{\tilde{A}_1}(\tilde{m})\mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m})\sigma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}) + \vartheta_{\tilde{B}_1}(\tilde{m}) - \vartheta_{\tilde{A}_1}(\tilde{m})\vartheta_{\tilde{B}_1}(\tilde{m}), \\ \phi_{\tilde{A}_1}(\tilde{m}) + \phi_{\tilde{B}_1}(\tilde{m}) - \phi_{\tilde{A}_1}(\tilde{m})\phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m}) + \gamma_{\tilde{B}_1}(\tilde{m}) - \gamma_{\tilde{A}_1}(\tilde{m})\gamma_{\tilde{B}_1}(\tilde{m}) \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} \mu_{\tilde{A}_2}(\tilde{n})\mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n})\sigma_{\tilde{B}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}) + \vartheta_{\tilde{B}_2}(\tilde{n}) - \vartheta_{\tilde{A}_2}(\tilde{n})\vartheta_{\tilde{B}_2}(\tilde{n}), \\ \phi_{\tilde{A}_2}(\tilde{n}) + \phi_{\tilde{B}_2}(\tilde{n}) - \phi_{\tilde{A}_2}(\tilde{n})\phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n}) + \gamma_{\tilde{B}_2}(\tilde{n}) - \gamma_{\tilde{A}_2}(\tilde{n})\gamma_{\tilde{B}_2}(\tilde{n}) \end{matrix} \right\rangle, \end{matrix}\right\}$$

Example 3.4:

Let $\tilde{\mathcal{P}} = \{\tilde{m}_1, \tilde{m}_2\}$ and $\tilde{\mathcal{Q}} = \{\tilde{n}_1\}$ be two universes and let

$\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right) = \left\{\begin{matrix} \langle(\tilde{m}_1, .34, .36, .22, .16, .71), (\tilde{m}_2, .63, .25, .5, .51, .2)\rangle, \\ \langle(\tilde{n}_1, .71, .11, .5, .3, .1)\rangle \end{matrix}\right\}$ and

$\left(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}\right) = \left\{\begin{matrix} \langle(\tilde{m}_1, .3, .7, .83, .42, .25), (\tilde{m}_2, .87, .7, .9, .68, .42)\rangle, \\ \langle(\tilde{n}_1, .91, .1, .9, .6, .72)\rangle \end{matrix}\right\}$ be two *PNBS*s in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then

$\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right) \otimes \left(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}\right) = \left\{\begin{matrix} \langle(\tilde{m}_1, .102, .252, .867, .512, .782), (\tilde{m}_2, .548, .175, .95, .843, .536)\rangle, \\ \langle(\tilde{n}_1, .646, .011, .95, .72, .748)\rangle \end{matrix}\right\}$

Definition 3.5

Let $\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right) = \left\{\begin{matrix} \langle\tilde{m}, \mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m})\rangle, \\ \langle\tilde{n}, \mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n})\rangle \end{matrix}\right\}$ be a *PNBS* in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then the *scalar multiplication* over a *PNBS* $\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right)$ denoted $\lambda_x.\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right)$ is defined as follows:

$$\lambda_x.\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right) = \left\{\begin{matrix} \langle 1 - \left(1 - \mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}, 1 - \left(1 - \sigma_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}, \vartheta_{\tilde{A}_1}(\tilde{m})^{\lambda_x}, \phi_{\tilde{A}_1}(\tilde{m})^{\lambda_x}, \gamma_{\tilde{A}_1}(\tilde{m})^{\lambda_x}\rangle, \\ \langle 1 - \left(1 - \mu_{\tilde{A}_2}(\tilde{n})\right)^{\lambda_x}, 1 - \left(1 - \sigma_{\tilde{A}_2}(\tilde{n})\right)^{\lambda_x}, \vartheta_{\tilde{A}_2}(\tilde{n})^{\lambda_x}, \phi_{\tilde{A}_2}(\tilde{n})^{\lambda_x}, \gamma_{\tilde{A}_2}(\tilde{n})^{\lambda_x}\rangle \end{matrix}\right\}$$

Example 3.6

Let $\tilde{\mathcal{P}} = \{\tilde{m}_1, \tilde{m}_2\}$ and $\tilde{\mathcal{Q}} = \{\tilde{n}_1, \tilde{n}_2\}$ be two universes and let

$\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right) = \left\{\begin{matrix} \langle(\tilde{m}_1, .28, .8, .24, .1, .7), (\tilde{m}_2, .8, .13, .7, .91, .3)\rangle, \\ \langle(\tilde{n}_1, .53, .6, .21, .19, .2), (\tilde{n}_2, .11, .13, .9, .7, .22)\rangle \end{matrix}\right\}$ be a *PNBS* in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$, and if $\lambda_x = 2$

Then, $\lambda_x \cdot \left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right) = \left\{\begin{matrix} \langle(\tilde{m}_1, .481, .96, .057, .01, .49), (\tilde{m}_2, .96, .243, .49, .828, .09)\rangle, \\ \langle(\tilde{n}_1, .779, .84, .044, .036, .04), (\tilde{n}_2, .207, .243, .81, .49, .048)\rangle \end{matrix}\right\}$

Definition 3.7

Let $\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right) = \left\{\begin{matrix} \langle\tilde{m}, \mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m})\rangle, \\ \langle\tilde{n}, \mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n})\rangle \end{matrix}\right\}$ be a *PNBS* in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then the *exponentiation* operation over a PNBS $\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right)$ denoted as $\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right)^{\lambda_x}$ and is defined as follows:

$$\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right)^{\lambda_x} = \left\{\begin{matrix} \langle \mu_{\tilde{A}_1}(\tilde{m})^{\lambda_x}, \sigma_{\tilde{A}_1}(\tilde{m})^{\lambda_x}, 1 - \left(1 - \vartheta_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}, 1 - \left(1 - \phi_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}, 1 - \left(1 - \gamma_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\rangle, \\ \langle \mu_{\tilde{A}_2}(\tilde{n})^{\lambda_x}, \sigma_{\tilde{A}_2}(\tilde{n})^{\lambda_x}, 1 - \left(1 - \vartheta_{\tilde{A}_2}(\tilde{n})\right)^{\lambda_x}, 1 - \left(1 - \phi_{\tilde{A}_2}(\tilde{n})\right)^{\lambda_x}, 1 - \left(1 - \gamma_{\tilde{A}_2}(\tilde{n})\right)^{\lambda_x}\rangle \end{matrix}\right\}$$

Example 3.8

Let $\tilde{\mathcal{P}} = \{\tilde{m}_1, \tilde{m}_2\}$ and $\tilde{\mathcal{Q}} = \{\tilde{n}_1, \tilde{n}_2\}$ be two universes and let

$\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right) = \left\{\begin{matrix} \langle(\tilde{m}_1, .28, .24, .7, .89, .9), (\tilde{m}_2, .79, .63, .37, .7, .8)\rangle, \\ \langle(\tilde{n}_1, .81, .74, .8, .4, .27), (\tilde{n}_2, .89, .9, .79, .15, .17)\rangle \end{matrix}\right\}$ be a *PNBS* in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$ and if $\lambda_x = 5$, then

$$\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right)^{\lambda_x} = \left\{\begin{matrix} \langle(\tilde{m}_1, .001, 0, .997, .999, .999), (\tilde{m}_2, .307, .099, .9, .997, .999)\rangle, \\ \langle(\tilde{n}_1, .348, .221, .999, .922, .792), (\tilde{n}_2, .558, .59, .999, .556, .606)\rangle \end{matrix}\right\}$$

Definition 3.9

Let $\left(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}\right) = \left\{\begin{matrix} \langle\mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m})\rangle, \\ \langle\mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n})\rangle \end{matrix}\right\}$ and

$(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle \mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{B}_1}(\tilde{m}) \rangle, \\ \langle \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{B}_2}(\tilde{n}), \vartheta_{\tilde{B}_2}(\tilde{n}), \phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{B}_2}(\tilde{n}) \rangle \end{Bmatrix}$ be two *PNBS*s in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then the *algebraic difference* of $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})$ and $(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ denoted as $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \ominus (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ and is defined as follows:

$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \ominus (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) =$

$$\begin{Bmatrix} \langle \frac{\mu_{\tilde{A}_1}(\tilde{m}) - \mu_{\tilde{B}_1}(\tilde{m})}{1-\mu_{\tilde{B}_1}(\tilde{m})}, \frac{\sigma_{\tilde{A}_1}(\tilde{m}) - \sigma_{\tilde{B}_1}(\tilde{m})}{1-\sigma_{\tilde{B}_1}(\tilde{m})}, \frac{\vartheta_{\tilde{A}_1}(\tilde{m})}{\vartheta_{\tilde{B}_1}(\tilde{m})}, \frac{\phi_{\tilde{A}_1}(\tilde{m})}{\phi_{\tilde{B}_1}(\tilde{m})}, \frac{\gamma_{\tilde{A}_1}(\tilde{m})}{\gamma_{\tilde{B}_1}(\tilde{m})} \rangle, \\ \langle \frac{\mu_{\tilde{A}_2}(\tilde{n}) - \mu_{\tilde{B}_2}(\tilde{n})}{1-\mu_{\tilde{B}_2}(\tilde{n})}, \frac{\sigma_{\tilde{A}_2}(\tilde{n}) - \sigma_{\tilde{B}_2}(\tilde{n})}{1-\sigma_{\tilde{B}_2}(\tilde{n})}, \frac{\vartheta_{\tilde{A}_2}(\tilde{n})}{\vartheta_{\tilde{B}_2}(\tilde{n})}, \frac{\phi_{\tilde{A}_2}(\tilde{n})}{\phi_{\tilde{B}_2}(\tilde{n})}, \frac{\gamma_{\tilde{A}_2}(\tilde{n})}{\gamma_{\tilde{B}_2}(\tilde{n})} \rangle \end{Bmatrix}$$

Which is true under given constraints $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \supseteq (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$, and the values of $\mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{B}_1}(\tilde{m}), \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{B}_2}(\tilde{n}) \neq 1$ and $\vartheta_{\tilde{B}_1}(\tilde{m})$, $\phi_{\tilde{B}_1}(\tilde{m})$, $\gamma_{\tilde{B}_1}(\tilde{m})$, $\vartheta_{\tilde{B}_2}(\tilde{n})$, $\phi_{\tilde{B}_2}(\tilde{n})$, $\gamma_{\tilde{B}_2}(\tilde{n}) \neq 0$.

Example 3.10:

Let $\tilde{\mathcal{P}} = \{\tilde{m}_1\}$ and $\mathcal{V} = \{\tilde{v}_1, \tilde{v}_2\}$ be two universes and let

$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \begin{Bmatrix} \langle (\tilde{m}_1, .7, .8, 0, .19, .28) \rangle, \\ \langle (\tilde{n}_1, .9, .72, .6, .55, .7), (\tilde{n}_2, .62, .11, .13, .32, .17) \rangle \end{Bmatrix}$ and

$(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle (\tilde{m}_1, .7, .1, .71, .2, .48) \rangle, \\ \langle (\tilde{n}_1, .25, .56, .8, .81, .76), (\tilde{n}_2, .6, .06, .2, .8, .19) \rangle \end{Bmatrix}$ be two *PNBS*s in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then

$$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \ominus (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle (\tilde{m}_1, 0, .777, 0, .95, .583) \rangle, \\ \langle (\tilde{n}_1, .866, .363, .75, .679, .921), (\tilde{n}_2, .05, .053, .65, .4, .894) \rangle \end{Bmatrix}$$

Definition 3.11

Let $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \begin{Bmatrix} \langle \mu_{\tilde{A}_1}(\tilde{m}), \sigma_{\tilde{A}_1}(\tilde{m}), \vartheta_{\tilde{A}_1}(\tilde{m}), \phi_{\tilde{A}_1}(\tilde{m}), \gamma_{\tilde{A}_1}(\tilde{m}) \rangle, \\ \langle \mu_{\tilde{A}_2}(\tilde{n}), \sigma_{\tilde{A}_2}(\tilde{n}), \vartheta_{\tilde{A}_2}(\tilde{n}), \phi_{\tilde{A}_2}(\tilde{n}), \gamma_{\tilde{A}_2}(\tilde{n}) \rangle \end{Bmatrix}$ and

$(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle \mu_{\tilde{B}_1}(\tilde{m}), \sigma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{B}_1}(\tilde{m}) \rangle, \\ \langle \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{B}_2}(\tilde{n}), \vartheta_{\tilde{B}_2}(\tilde{n}), \phi_{\tilde{B}_2}(\tilde{n}), \gamma_{\tilde{B}_2}(\tilde{n}) \rangle \end{Bmatrix}$ be two *PNBS*s on $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then the *algebraic quotient* of $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})$ and $(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ denoted as $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \oplus (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ and is defined as follows:

$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \oplus (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) =$

$$\begin{Bmatrix} \langle \frac{\mu_{\tilde{A}_1}(\tilde{m})}{\mu_{\tilde{B}_1}(\tilde{m})}, \frac{\sigma_{\tilde{A}_1}(\tilde{m})}{\sigma_{\tilde{B}_1}(\tilde{m})}, \frac{\vartheta_{\tilde{A}_1}(\tilde{m}) - \vartheta_{\tilde{B}_1}(\tilde{m})}{1-\vartheta_{\tilde{B}_1}(\tilde{m})}, \frac{\phi_{\tilde{A}_1}(\tilde{m}) - \phi_{\tilde{B}_1}(\tilde{m})}{1-\phi_{\tilde{B}_1}(\tilde{m})}, \frac{\gamma_{\tilde{A}_1}(\tilde{m}) - \gamma_{\tilde{B}_1}(\tilde{m})}{1-\gamma_{\tilde{B}_1}(\tilde{m})} \rangle, \\ \langle \frac{\mu_{\tilde{A}_2}(\tilde{n})}{\mu_{\tilde{B}_2}(\tilde{n})}, \frac{\sigma_{\tilde{A}_2}(\tilde{n})}{\sigma_{\tilde{B}_2}(\tilde{n})}, \frac{\vartheta_{\tilde{A}_2}(\tilde{n}) - \vartheta_{\tilde{B}_2}(\tilde{n})}{1-\vartheta_{\tilde{B}_2}(\tilde{n})}, \frac{\phi_{\tilde{A}_2}(\tilde{n}) - \phi_{\tilde{B}_2}(\tilde{n})}{1-\phi_{\tilde{B}_2}(\tilde{n})}, \frac{\gamma_{\tilde{A}_2}(\tilde{n}) - \gamma_{\tilde{B}_2}(\tilde{n})}{1-\gamma_{\tilde{B}_2}(\tilde{n})} \rangle \end{Bmatrix}$$

Which is true under given constraints $(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) \supseteq (\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})$, where the values of $\mu_{\tilde{B}_1}(\tilde{m})$, $\sigma_{\tilde{B}_1}(\tilde{m}), \mu_{\tilde{B}_2}(\tilde{n}), \sigma_{\tilde{B}_2}(\tilde{n}) \neq 0$ and $\vartheta_{\tilde{B}_1}(\tilde{m}), \phi_{\tilde{B}_1}(\tilde{m}), \gamma_{\tilde{B}_1}(\tilde{m}), \vartheta_{\tilde{B}_2}(\tilde{n})$, $\phi_{\tilde{B}_2}(\tilde{n})$, $\gamma_{\tilde{B}_2}(\tilde{n}) \neq 1$.

Example 3.12

Let $\tilde{\mathcal{P}} = \{\tilde{m}_1, \tilde{m}_2\}$ and $\tilde{\mathcal{Q}} = \{\tilde{n}_1, \tilde{n}_2\}$ be two universes and let

$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) = \begin{Bmatrix} \langle (\tilde{m}_1, .1, .3, .66, .75, .5), (\tilde{m}_2, .09, .09, .911, .2, .92) \rangle, \\ \langle (\tilde{n}_1, .1, .11, .33, .7, .81), (\tilde{n}_2, .25, .25, .42, .7, .61) \rangle \end{Bmatrix}$ and

$(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle (\tilde{m}_1, .2, .4, .56, .7, .21), (\tilde{m}_2, .1, .11, .9, .1, .71) \rangle, \\ \langle (\tilde{n}_1, .2, .22, .222, .5, .56), (\tilde{n}_2, .3, .33, .333, .59, .27) \rangle \end{Bmatrix}$ be two *PNBS*s in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$. Then $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \oplus$ $(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = \begin{Bmatrix} \langle (\tilde{m}_1, .5, .75, .227, .166, .367), (\tilde{m}_2, .9, .818, .11, .111, .724) \rangle, \\ \langle (\tilde{n}_1, .5, .5, .138, .4, .568), (\tilde{n}_2, .833, .757, .13, .268, .465) \rangle \end{Bmatrix}$

Theorem 3.13

Let $(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})$ and $(\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2})$ be two *PNBS*s in $\tilde{\mathcal{P}}$ and $\tilde{\mathcal{Q}}$ and $\lambda_x, \lambda_y, \lambda_z > 0$. Then

$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \oplus (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) \oplus (\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})$

$(\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2}) \otimes (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) = (\tilde{\mathcal{R}}_{B_1}, \tilde{\mathcal{R}}_{B_2}) \otimes (\tilde{\mathcal{R}}_{A_1}, \tilde{\mathcal{R}}_{A_2})$

$\lambda_x\left(\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)\right)=\lambda_x.\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\lambda_x.\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)$

$$\lambda_x.\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\lambda_y.\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)=\left(\lambda_x\oplus\lambda_y\right).\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)$$

$\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)^{\lambda_x}\otimes\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)^{\lambda_y}=\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)^{\lambda_x+\lambda_y}$

$\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)^{\lambda_x}\otimes\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)^{\lambda_x}=\left(\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\otimes\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)\right)^{\lambda_x}$

It is easy to prove the results (a) and (b); and hence we prove the others results

Proof (c)

By definition, we have

$\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)=$

$$\left\{\begin{array}{c}\left\langle\begin{array}{c}\mu_{\tilde{A}_1}(\tilde{m})+\mu_{\tilde{B}_1}(\tilde{m})-\mu_{\tilde{A}_1}(\tilde{m})\mu_{\tilde{B}_1}(\tilde{m}),\sigma_{\tilde{A}_1}(\tilde{m})+\sigma_{\tilde{B}_1}(\tilde{m})-\sigma_{\tilde{A}_1}(\tilde{m})\sigma_{\tilde{B}_1}(\tilde{m}),\\ \vartheta_{\tilde{A}_1}(\tilde{m})\vartheta_{\tilde{B}_1}(\tilde{m}),\phi_{\tilde{A}_1}(\tilde{m})\phi_{\tilde{B}_1}(\tilde{m}),\gamma_{\tilde{A}_1}(\tilde{m})\gamma_{\tilde{B}_1}(\tilde{m})\end{array}\right\rangle,\\ \left\langle\begin{array}{c}\mu_{\tilde{A}_2}(\tilde{n})+\mu_{\tilde{B}_2}(\tilde{n})-\mu_{\tilde{A}_2}(\tilde{n})\mu_{\tilde{B}_2}(\tilde{n}),\sigma_{\tilde{A}_2}(\tilde{n})+\sigma_{\tilde{B}_2}(\tilde{n})-\sigma_{\tilde{A}_2}(\tilde{n})\sigma_{\tilde{B}_2}(\tilde{n}),\\ \vartheta_{\tilde{A}_2}(\tilde{n})\vartheta_{\tilde{B}_2}(\tilde{n}),\phi_{\tilde{A}_2}(\tilde{n})\phi_{\tilde{B}_2}(\tilde{n}),\gamma_{\tilde{A}_2}(\tilde{n})\gamma_{\tilde{B}_2}(\tilde{n})\end{array}\right\rangle\end{array}\right\}$$

$\lambda_x\left(\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)\right)=$

$$\left\{\begin{array}{c}\left\langle\begin{array}{c}1-\left(1-\left(\mu_{\tilde{A}_1}(\tilde{m})+\mu_{\tilde{B}_1}(\tilde{m})-\mu_{\tilde{A}_1}(\tilde{m})\mu_{\tilde{B}_1}(\tilde{m})\right)\right)^{\lambda_x},1-\left(1-\left(\sigma_{\tilde{A}_1}(\tilde{m})+\sigma_{\tilde{B}_1}(\tilde{m})-\sigma_{\tilde{A}_1}(\tilde{m})\sigma_{\tilde{B}_1}(\tilde{m})\right)\right)^{\lambda_x},\\ \left(\vartheta_{\tilde{A}_1}(\tilde{m})\vartheta_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x},\left(\phi_{\tilde{A}_1}(\tilde{m})\phi_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x},\left(\gamma_{\tilde{A}_1}(\tilde{m})\gamma_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\end{array}\right\rangle,\\ \left\langle\begin{array}{c}1-\left(1-\left(\mu_{\tilde{A}_2}(\tilde{n})+\mu_{\tilde{B}_2}(\tilde{n})-\mu_{\tilde{A}_2}(\tilde{n})\mu_{\tilde{B}_2}(\tilde{n})\right)\right)^{\lambda_x},1-\left(1-\left(\sigma_{\tilde{A}_2}(\tilde{n})+\sigma_{\tilde{B}_2}(\tilde{n})-\sigma_{\tilde{A}_2}(\tilde{n})\sigma_{\tilde{B}_2}(\tilde{n})\right)\right)^{\lambda_x},\\ \left(\vartheta_{\tilde{A}_2}(\tilde{n})\vartheta_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x},\left(\phi_{\tilde{A}_2}(\tilde{n})\phi_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x},\left(\gamma_{\tilde{A}_2}(\tilde{n})\gamma_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x}\end{array}\right\rangle\end{array}\right\}$$

Now we solve the truth membership functions

$$\begin{aligned}1-\left(1-\left(\mu_{\tilde{A}_1}(\tilde{m})+\mu_{\tilde{B}_1}(\tilde{m})-\mu_{\tilde{A}_1}(\tilde{m})\mu_{\tilde{B}_1}(\tilde{m})\right)\right)^{\lambda_x}&=1-\left(1-\mu_{\tilde{A}_1}(\tilde{m})-\mu_{\tilde{B}_1}(\tilde{m})+\mu_{\tilde{A}_1}(\tilde{m})\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\\ &=1-\left(\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)-\mu_{\tilde{B}_1}(\tilde{m})\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)\right)^{\lambda_x}\\ &=1-\left(\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)\right)^{\lambda_x}\end{aligned}$$

By similar argument, we get

$\lambda_x\left(\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)\right)=$

$$\left\{\begin{array}{c}\left\langle\begin{array}{c}1-\left(\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)\right)^{\lambda_x},1-\left(\left(1-\sigma_{\tilde{A}_1}(\tilde{m})\right)\left(1-\sigma_{\tilde{B}_1}(\tilde{m})\right)\right)^{\lambda_x},\\ \left(\vartheta_{\tilde{A}_1}(\tilde{m})\vartheta_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x},\left(\phi_{\tilde{A}_1}(\tilde{m})\phi_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x},\left(\gamma_{\tilde{A}_1}(\tilde{m})\gamma_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\end{array}\right\rangle,\\ \left\langle\begin{array}{c}1-\left(\left(1-\mu_{\tilde{A}_2}(\tilde{n})\right)\left(1-\mu_{\tilde{B}_2}(\tilde{n})\right)\right)^{\lambda_x},1-\left(\left(1-\sigma_{\tilde{A}_2}(\tilde{n})\right)\left(1-\sigma_{\tilde{B}_2}(\tilde{n})\right)\right)^{\lambda_x},\\ \left(\vartheta_{\tilde{A}_2}(\tilde{n})\vartheta_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x},\left(\phi_{\tilde{A}_2}(\tilde{n})\phi_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x},\left(\gamma_{\tilde{A}_2}(\tilde{n})\gamma_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x}\end{array}\right\rangle\end{array}\right\} \quad \ldots\ldots..(1)$$

Now, $\lambda_x.\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\lambda_x.\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)=$

$$\left\{\begin{array}{c}\left\langle\begin{array}{c}\left(1-\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\right)+\left(1-\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\right)-\left(1-\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\right)\left(1-\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\right),\\ \left(1-\left(1-\sigma_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\right)+\left(1-\left(1-\sigma_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\right)-\left(1-\left(1-\sigma_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\right)\left(1-\left(1-\sigma_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\right),\\ \vartheta_{\tilde{A}_1}(\tilde{m})^{\lambda_x}\vartheta_{\tilde{B}_1}(\tilde{m})^{\lambda_x},\phi_{\tilde{A}_1}(\tilde{m})^{\lambda_x}\phi_{\tilde{B}_1}(\tilde{m})^{\lambda_x},\gamma_{\tilde{A}_1}(\tilde{m})^{\lambda_x}\gamma_{\tilde{B}_1}(\tilde{m})^{\lambda_x}\end{array}\right\rangle,\\ \left\langle\begin{array}{c}\left(1-\left(1-\mu_{\tilde{A}_2}(\tilde{n})\right)^{\lambda_x}\right)+\left(1-\left(1-\mu_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x}\right)-\left(1-\left(1-\mu_{\tilde{A}_2}(\tilde{n})\right)^{\lambda_x}\right)\left(1-\left(1-\mu_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x}\right),\\ \left(1-\left(1-\sigma_{\tilde{A}_2}(\tilde{n})\right)^{\lambda_x}\right)+\left(1-\left(1-\sigma_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x}\right)-\left(1-\left(1-\sigma_{\tilde{A}_2}(\tilde{n})\right)^{\lambda_x}\right)\left(1-\left(1-\sigma_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x}\right),\\ \vartheta_{\tilde{A}_2}(\tilde{n})^{\lambda_x}\vartheta_{\tilde{B}_2}(\tilde{n})^{\lambda_x},\phi_{\tilde{A}_2}(\tilde{n})^{\lambda_x}\phi_{\tilde{B}_2}(\tilde{n})^{\lambda_x},\gamma_{\tilde{A}_2}(\tilde{n})^{\lambda_x}\gamma_{\tilde{B}_2}(\tilde{n})^{\lambda_x}\end{array}\right\rangle\end{array}\right\}$$

Now we solve the truth membership functions

$$\left(1-\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\right)+\left(1-\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\right)-\left(1-\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\right)\left(1-\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\right)$$

$$=2-\left[\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}+\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\right]$$
$$-\left[1-\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}-\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}+\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\right]$$

$$=2-\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}-\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}-1+\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}+\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}$$
$$-\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}$$

$$=1-\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)^{\lambda_x}\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}$$

By similar argument, we get

$$\lambda_x.\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\lambda_x.\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)=$$

$$\left\{\begin{array}{c}\left\langle\begin{array}{c}1-\left(\left(1-\mu_{\tilde{A}_1}(\tilde{m})\right)\left(1-\mu_{\tilde{B}_1}(\tilde{m})\right)\right)^{\lambda_x},1-\left(\left(1-\sigma_{\tilde{A}_1}(\tilde{m})\right)\left(1-\sigma_{\tilde{B}_1}(\tilde{m})\right)\right)^{\lambda_x},\\ \left(\vartheta_{\tilde{A}_1}(\tilde{m})\vartheta_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x},\left(\phi_{\tilde{A}_1}(\tilde{m})\phi_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x},\left(\gamma_{\tilde{A}_1}(\tilde{m})\gamma_{\tilde{B}_1}(\tilde{m})\right)^{\lambda_x}\end{array}\right\rangle,\\ \left\langle\begin{array}{c}1-\left(\left(1-\mu_{\tilde{A}_2}(\tilde{n})\right)\left(1-\mu_{\tilde{B}_2}(\tilde{n})\right)\right)^{\lambda_x},1-\left(\left(1-\sigma_{\tilde{A}_2}(\tilde{n})\right)\left(1-\sigma_{\tilde{B}_2}(\tilde{n})\right)\right)^{\lambda_x},\\ \left(\vartheta_{\tilde{A}_2}(\tilde{n})\vartheta_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x},\left(\phi_{\tilde{A}_2}(\tilde{n})\phi_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x},\left(\gamma_{\tilde{A}_2}(\tilde{n})\gamma_{\tilde{B}_2}(\tilde{n})\right)^{\lambda_x}\end{array}\right\rangle\end{array}\right\}\ldots\ldots\ldots(2)$$

From (1) and (2), we get

$$\lambda_x\left(\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)\right)=\lambda_x.\left(\tilde{\mathcal{R}}_{A_1},\tilde{\mathcal{R}}_{A_2}\right)\oplus\lambda_x.\left(\tilde{\mathcal{R}}_{B_1},\tilde{\mathcal{R}}_{B_2}\right)$$

Proof (d), (e) and (f):

By the above similar arguments, we prove the others.

4. CONCLUSION

This paper presents various algebraic operations on pentapartitioned neutrosophic binary set to handle problems requiring dual domains such as data fusion, cross domain optimization, etc. The proposed operations, including algebraic sum, algebraic product, scalar multiplication, exponentiation, algebraic difference, and algebraic quotient were defined and their properties, such as commutativity under algebraic sum and algebraic product, distributivity of scalar multiplication over algebraic sum as well as scalar addition, algebraic product of exponentiation, and power of an algebraic product were mathematically proven. By introducing these operations, we have addressed the scenarios where conventional single-universe approaches are inadequate. The topics of further research include applying pentapartitioned neutrosophic binary set in various algebraic structures, extending these operations to multiple domains, and exploring their integration with other uncertainty models like bipolar soft sets, hyper soft sets, etc.

REFERENCES

[1] Anit Yoha. A and Jaslin Melbha. M, *Pentapartitioned Neutrosophic Binary Set And Its Properties,* "International Journal of Neutrosophic Science", vol. 24, No.02, pp. 237-245, 2024.

[2] Atanassov, K.T. *Intuitionistic fuzzy sets*. Fuzzy Sets Syst. 1986, 20, 87–96.

[3] Atanassov, K.T.; Gargov, G. *Interval valued intuitionistic fuzzy sets*. Fuzzy Sets Syst. 1989, 31,343-349.

[4] Atanassov, K.T.; Riecan, B. *On two operations over intuitionistic fuzzy sets*. J. Appl. Math. Stat. Inform. 2006,2, 145–148.

[5] Atanassov, K.T. *Remark on operations "subtraction" over intuitionistic fuzzy sets*. Notes Intuit. Fuzzy Sets2009, 15, 20–24.

[6] Atanassov, K.T. *On Intuitionistic Fuzzy Sets Theory*; Springer: Berlin, Germany, 2012.

[7] Atanassova, L. *A new operator over intitionistic fuzzy sets. Notes Intuit. Fuzzy Sets* 2020, 26, 23–27.

[8] Chen, T.Y. *Remarks on the subtraction and division operations over intuitionistic fuzzy sets and interval-valued fuzzy sets*. Int. J. Fuzzy Syst 2007, 9, 169–172.

[9] Hazwani Hashim, L. Abdullah, and A. Al-Quran, "*Interval Neutrosophic Vague Sets*," 2019.

[10] Hazwani Hashim, Lazim Abdullah and Ashraf Al-Quran, *"Algebraic operations on new interval neutrosophic vague sets*, ICoAIMS, IOP Publishing, Journal of Physics: Conference Series 1366 (2019) 012062", 2019.

[11] Karaaslan and Hayat. K, "*Some new operations on single-valued neutrosophic matrices and their applications in multi-criteria group decision making*," Appl. Intell., vol. 48, no. 12, pp. 4594–4614, 2018.

[12] Lu Z.K.; Ye, J. *Decision-making method for clay-brick selection based on subtraction operational aggregation operators of intuitionistic fuzzy values*. Open Cybern. Syst. J. 2016, 10, 283–291.

[13] Liu. P, Wang. Y (2014), *Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean*, Neural Comput Appl 25:2001–2010

[14] Peng. X and Dai. J, "*A bibliometric analysis of neutrosophic set: two decades review from1998 to 2017*," Artificial Intelligence Review, Springer Netherlands, pp. 1–57, 18-Aug-2018.

[15] Radhika V. R, Mohana K (2024), *Algebraic operations on pentapartitioned neutrosophic vague sets*, Recent Trends in Fuzzy Set Theory and its Applications. 10.58532/nbennurch280.

[16] Rama Malik and Surapati Pramanik, *Pentapartitioned neutrosophic set and its properties,* Neutrosophic Sets and Systems, vol. 36, 2020.

[17] Remya P. B., Francina Shalini A., *Neutrosophic Vague Binary Set.* Neutrosophic Sets and Systems, 29, (1) 2019.

[18] Rivieccio. U, "*Neutrosophic logics: Prospects and problems*," Fuzzy Sets Syst., vol. 159, no. 14, pp. 1860–1868, 2008.

[19] Smarandache. F, *A unifying field of logics: Neutrosophy, neutrosophic probability, set and logic,* American Research Press, Rehoboth, 1998.

[20] Smarandache. F, *Neutrosophy, Neutrosophic Probability, Set and Logic,* American Rescue Press, Rehoboth, DE, USA, 1998.

[21] Smarandache. F, *Subtraction and division of neutrosophic numbers*, Crit. Rev,13, 103–110, 2016.

[22] Surekha. S. S, Elekiah. J and Sindhu. G, *A study on Neutrosophic Binary Topological space,* Stochastic Modelling and applications, Vol 26(3), 479-486.

[23] Wang Haibin, F. Smarandache, Y.-Q. Zhang, and R. Sunderraman, *"Interval Neutrosophic Sets and Logic: Theory and Applications in Computing,"* in Hexis, Phoenix, 2005.

[24] Wang Haibin, Smarandache, Zhang Yanqing, Sunderraman Rajshekhar; (2012); *Single Valued Neutrosophic Sets;* 10.

[25] Ye, J. *A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets*.J. Intell. Fuzzy Syst. 2014, 26, 2459–2466.

[26] Ye. J, "*Trapezoidal neutrosophic set and its application to multiple attribute decision- making"*, Neural Comput. Appl., vol.26, no.5, pp. 1157-1166, Jul. 2015.

[27] Ye. J; *Subtraction and Division Operations of Simplified Neutrosophic Sets*, Information. 8. 51. 10.3390/info8020051, 2017.

[28] Zadeh. L.A, *Fuzzy sets*, Information and Control, 8 (3), 338–353, 1965

[29] Zhang, H.Y.; Wang, J.Q.; Chen, X.H. Interval neutrosophic sets and their application in multicriteria decisionmaking problems. Sci. World J. 2014.