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Group Transformations technique for Non-Newtonian Natural Convection Boundary Layers

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ABSTRACT
This study is focused on the incompressible natural convection boundary layer flow of non-Newtonian fluid. The governing differential equations are transformed to a system of non-linear ordinary differential equation by Scaling transformation technique and solved numerically, solution is obtained for particular non-Newtonian Sutterby fluid model. The important physical quantity like velocity and temperature distribution of the Sutterby fluids, local skin friction C_f, Nusselt number N_u are discussed.

Keywords: *Scaling transformation technique, Boundary layer, Sutterby fluids, Natural convection, non-Newtonian.*

1. INTRODUCTION

Natural convection is a phenomenon with significant technological and scientific relevance. It plays a critical role in applications like cooling nuclear reactors and serving as heat sinks in turbine blades. Furthermore, thermal convection profoundly impacts the internal structures of stars and planets, shaping their dynamics and evolution. The concept of similarity transformation has been explored by several authors. Similarity transformation is used to transform the set of partial differential equations to a system of ordinary differential equations and the solution of the resulting ordinary differential equation is known as the similarity solution.

Voller [1] presented a comprehensive report on one dimensional solidification of an under cooled binary alloy using a similarity solution. Timol and Kalthia [2] is probably first to develop systematic analysis of natural convection flows of all non-Newtonian visco inelastic fluids characterized by the special functional relationship of stress strain components. Darji R.M. and Timol M.G. [3] analysed non-Newtonian fluid flow of heat transfer in natural convection boundary layer flow of the generalized non-Newtonian fluid over a vertical flat plate by using deductive group symmetry method. Similarity solution the heat conduction considering the cylindrical channel analysed by Zhou [4]. Forced convective flow and heat transfer over a porous plate in a Darcy-Forchheimer porous medium in presence of radiation investigated by Mukhopadhya [5]. The phenomena of unsteady magnetohydrodynamics (MHD) natural convection flow in an inclined square cavity filled with nanofluid and containing a heated circular obstacle at its center with heat generation/absorption impact investigated numerically by Mansour M.A., Gorla R.S., Siddiqa S.,[6]. Natural convection in a triangular enclosure filled with a porous medium saturated with Cu–water nanofluid studied by S. E. Ahmed, A. M. Rashad, and R. S. R. Gorla [7]. The natural convection boundary layer flow of some non-Newtonian Visco inelastic fluids over a vertical flate plate analysed by Sonawane P., Timol M.G., Salunke J.N., [8]. The laminar natural convection of a non-Newtonian fluid along a vertical isothermal surface with the boundary-layer equations for a Sutterby fluid are solved numerically, and several characteristics of the non-similarity solution represented by Fujii [9].

In the scientific literature, a significant body of research is available on the topic of natural convective heat transfer from vertical isothermal surfaces to non-Newtonian fluids. This area of study is particularly important because non-Newtonian fluids, which exhibit flow behaviour that deviates from the standard Newtonian model are commonly encountered in various industrial and natural processes. Nita Jain, R.M. Darji, M.G. Timol [10] derived Similarity solution of natural convection boundary layer flow of non-Newtonian Sutterby fluids. So motivated by these, we studied in the present paper using Scaling group transformation technique boundary layer analysis of an incompressible natural convection in non-Newtonian fluid flow.

2. MATHEMATICAL FORMATION

The governing equations for continuity, momentum, and heat transfer are formulated for a two-dimensional, steady, incompressible, and laminar natural convection flow along a vertical flat plate using a Cartesian coordinate system with standard notations and introducing the stream function ψ , the continuity equation become identical and momentum and energy equation as,

$$\psi_y^* \psi_{xy}^* - \psi_x^* \psi_{yy}^* = \left(\tau_{yx}^*\right)_y + \theta^* \tag{1}$$

$$3P_r(\psi_y^* \theta_x^* - \psi_x^* \theta_y^*) = \theta_{yy}^* \tag{2}$$

$$\mathfrak{L}\left(\tau^*{}_{yx}, \psi_{yy}^*\right) = 0 \tag{3}$$

With boundary conditions,

$$y = 0, \quad \psi_y^* = \psi_x^* = 0, \quad \theta^* = \theta_w \tag{4}$$

$$y = \infty, \quad \psi_y^* = 0\,, \quad \theta^* = 0 \tag{5}$$

Group Theoretic Method

Group theoretic method which is used to find the similarity transformation is based on the concepts derived from continuous transformation of groups. For the present problem we introduce one parameter group of transformation given below

$$\bar{x}^* = \mathfrak{P}^{\beta_1} x^*\ , \quad \bar{y}^* = \mathfrak{P}^{\beta_2} y^*, \quad \bar{\tau}_{yx}^* = \mathfrak{P}^{\beta_3} \tau_{yx}^*$$

$$\bar{\psi}^* = \mathfrak{P}^{\beta_4} \psi^*\ , \quad \bar{\theta}^* = \mathfrak{P}^{\beta_5} \theta^* \tag{6}$$

Where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ and $\mathfrak{P}$ are constants

From equation (6) one obtains

$$\left(\frac{\bar{x}^*}{x^*}\right)^{\frac{1}{\beta_1}} = \left(\frac{\bar{y}^*}{y^*}\right)^{\frac{1}{\beta_2}} = \left(\frac{\bar{\tau}_{yx}^*}{\tau_{yx}^*}\right)^{\frac{1}{\beta_3}} = \left(\frac{\bar{\psi}^*}{\psi^*}\right)^{\frac{1}{\beta_4}} = \left(\frac{\bar{\theta}^*}{\theta^*}\right)^{\frac{1}{\beta_5}} = \mathfrak{P} \tag{7}$$

Introducing the linear transformation, given by equation (7), into the Equations (1-3) and solving the differential equation are completely invariant to the proposed linear transformation, the following coupled algebraic equations and solving these equations with$\beta_1 \neq 0$, we obtain

$$\beta_2 = \frac{1}{3}\ \beta_1\ , \quad \beta_3 = 0, \quad \beta_4 = \frac{2}{3}\ \beta_1\ , \quad \beta_5 = -\frac{1}{3}\beta_1 \tag{8}$$

Introducing equation (8) into equation (7) result in

$$\eta = \frac{y^*}{x^{*\frac{1}{3}}}\ , \quad \psi^* = F(\eta) x^{*\frac{2}{3}}, \quad \theta^* = F_1(\eta) x^{*-\frac{1}{3}} \quad and \quad \tau_{yx}^* = F_2(\eta) \tag{9}$$

With the boundary conditions, equation (4-5) becomes

$$\eta = 0\ , \quad F_1(0) = 1 \tag{10}$$

$$\eta \to \infty\ , \quad F_1(\infty) = 0 \tag{11}$$

Introducing equations (9) in equation (1)-(3), we get following ordinary differential equations

$$F'^2(\eta) + 2F(\eta)F''(\eta) - 3F_2'(\eta) - 3F_1(\eta) = 0 \tag{12}$$

$$2F(\eta)F_1'(\eta) + F'(\eta)F_1(\eta) + \frac{1}{Pr} F_1''(\eta) = 0 \tag{13}$$

With stress-strain relationship is given by,

$$\mathfrak{L}(F_2, F'') = 0 \tag{14}$$

With the boundary condition

$$\eta = 0\ , \quad F'(0) = 0\,, F(0) = 0\ \,, F_1(0) = 1$$

$$\eta \to \infty\,, \quad F'(\infty) = 0, \quad F_1(\infty) = 0 \tag{15}$$

Numerical Methodology

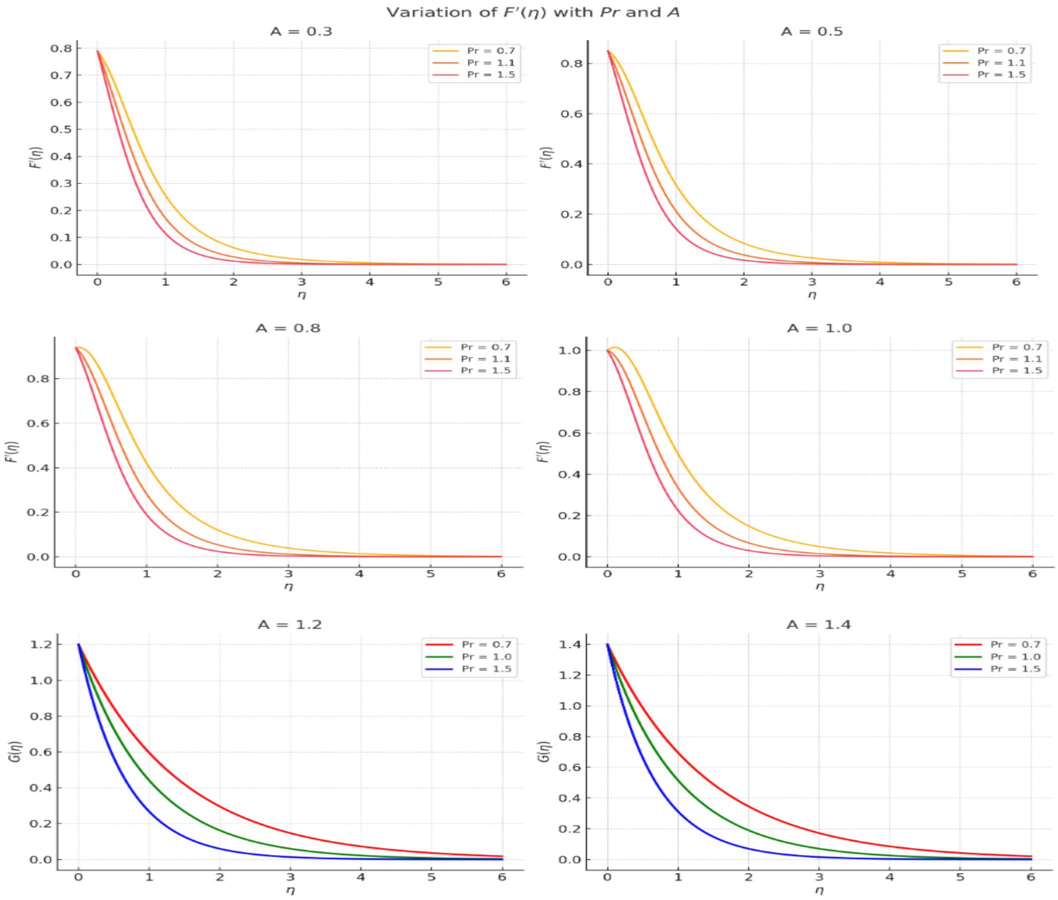
The main focus is the Sutterby fluid model is considered as

$$\tau_{yx} = \mu_0 \left[\frac{\sin^{-1}\left(B\frac{\partial u}{\partial y}\right)}{\left(B\frac{\partial u}{\partial y}\right)}\right]^A \frac{\partial u}{\partial y} \tag{16}$$

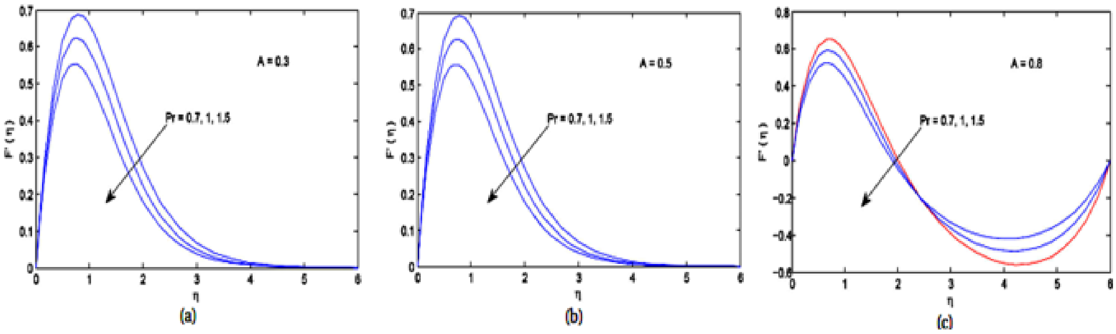
Introducing a dimensionless quantity and applying similarity variables is a common approach in mathematical modelling to simplify equations, especially partial differential equations.

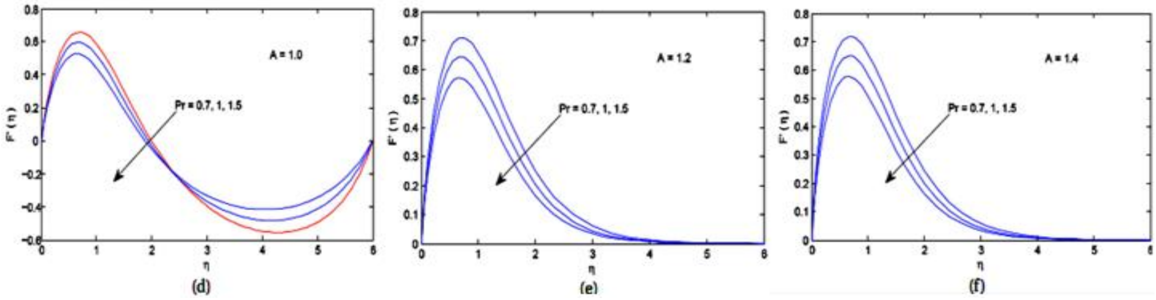
- The equation is solved numerical method as described by Nachtsheim and Swigert [11].
- **The integration is carried out with the following settings:**
- Starting at $\eta = 0$ and $h = 0.5$, integration is performed until η_{stop}=10
- Fixed non-dimensional numbers: $\alpha = 10$, β=1.5×10^{-2} and varying Pr = 0.7, 1.0, 1.5
- Velocity profiles F′(η) and Temperature profiles $G(\eta)$ are plotted for these Pr values.
- Each graph plots G(η) for different Prandtl numbers (Pr = 0.7, 1, 1.5). These graphs show how G(η) changes with η for the given Prandtl numbers and values of A=0.3,0.5,0.8,1.0,1.2,1.4.
- We plot the graphs for Velocity profile and Temperature profile for varies A

1) Graphs for velocity profile F′(η) with respect to similarity variable η

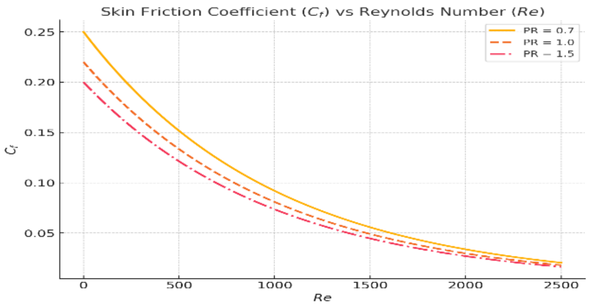


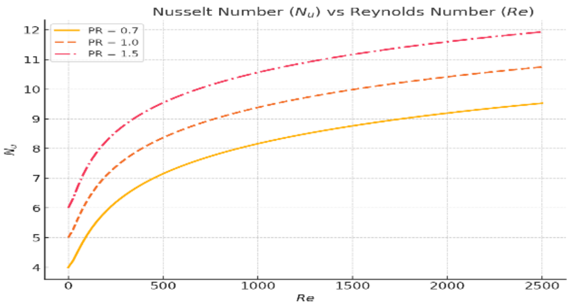
2) Graphs for Temperature profile $G(\eta)$ with respect to similarity variable η





3) Graphs for Skin friction coefficient C_f and Nusselt number N_u w.r.t to Reynolds Number Re





3. CONCLUSION

The results are visually presented in the form of graphs, which effectively illustrate the velocity and temperature profiles of the fluid for for various values of the flow consistency index shown in graphs. Skin Friction Coefficient C_f with respect to Reynolds Number Re.

- Shows the decreasing trend of Skin Friction Coefficient C_f as Reynolds Number Re increases.
- Higher Prandtl numbers Pr slightly reduce C_f indicating reduced shear forces and Demonstrates increasing Nusselt number N_u with Higher Reynolds Number Re.

The findings demonstrate a good agreement with the solutions presented by Timol M.G.et al. [10]

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