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Investigation of Hydromagnetic flow and heat transfer in a Boussinesq - Stokes suspension over an exponentially stretching sheet

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ABSTRACT

The investigation of velocity profiles and heat transfer properties in a hydromagnetic Boussinesq-Stokes suspension (BSS) flow over an exponentially stretching impermeable sheet is presented in this paper. Partial differential equations are the fundamental formulas that control flow and heat transfer. A suitable local similarity transformation has been applied to convert the equations into nonlinear ordinary differential equations. The differential transform method (DTM) is used to obtain the series solutions of the transformed equations with guaranteed convergence. On velocity profiles and heat transfer, the influence of the Chandrasekhar number, couple stress parameter, Prandtl number, and Eckert number are examined. The conclusions are explained graphically.

Keywords: *MHD, Chandrasekhar Number, Differential Transform Method, Couple stress, Prandtl number, Eckert number.*

1. INTRODUCTION

There are numerous uses for boundary layer flow on a continually stretched sheet, wire drawing, including hot rolling, the manufacture of glass fibre, and the making of paper. Numerous papers have been published on different elements of flow characteristics in boundary layer flow across a stretching boundary in Newtonian fluids since Crane's groundbreaking theoretical work [1970]. The majority of these studies focus on boundary layer flows over stretching surfaces, where it is considered that the surface's velocity stretches in quadratic proportion to the distance from the static origin, as developed by Kumaran and Ramanaiah [1996], or in linear proportion, as developed by Crane [1970].

It is well known that the liquids around the stretching sheet help the sheet cool at a suitable rate, and that this has a significant impact on the properties of the finished product. Siddheshwar and Mahabaleshwar's paper [2005] provide a good explanation on this work. In practical terms, stretching cannot be expressed as an even or linear quadratic function of the axial coordinate except the procedure is carried out exactly.

Regarding this, it is necessary to investigate the flow produced by exponential stretching in the axial coordinate. For the first instance, the mass and heat transfer in a boundary layer flow caused by an exponentially stretching continuous surface was investigated by Magyari and Keller (1999). The exponential stretching sheet problem with suction was examined by Elbashbeshy (2001).

Numerous studies on the exponentially stretching sheet problem have surfaced, taking into account different [see Partha *et al.* (2005), Khan and Sanjayanand (2005), Al-Odat *et al* (2006), Sanjayanand and Khan (2006), Chen *et al.* (2006), Sajid and Hayat (2008), Rashidi and Keimanesh (2010), Ishak (2011), Haas and Oliveski (2011), Siddheshwar *et al* (2014)].

Most of the aforementioned works either employ a challenging homotopy analysis method or a first order solution. For the previously stated reasons, we were opted to investigate the stretching sheet problem including a BSS in magnetohydrodynamics.

Driven by the studies mentioned above and potential uses, we plan to examine heat transfer and momentum in a BSS flow across an exponentially expanding sheet. To get a series solution, the DTM is applied.

The solution for temperature is obtained for non-isothermal boundary conditions of two the kinds (i) prescribed exponential order boundary heat flux (PEHF) and (ii) prescribed exponential order surface temperature (PEST).

2. NOMENCLATURE

C couple stress parameter

E Eckert number $\left(\frac{\alpha^2 l^2}{AC_p}\right)$

C_p specific heat at constant pressure

F dimensionless stream function

Pr Prandtl number

E_s scaled Eckert number $\left(\frac{E}{D}\sqrt{\frac{\alpha}{\nu}}\right)$

H_0 uniform magnetic field

Q Chandrasekhar number

L reference length

Y dimensionless vertical coordinate

Re Reynold's number T

T temperature of liquid

T_w wall (sheet) temperature

T_∞ liquid temperature far away from the sheet

U dimensionless horizontal velocity

u dimensional horizontal velocity

U_0 characteristic velocity

X dimensionless horizontal coordinate

v dimensional vertical velocity

θ non-dimensional temperature

V dimensionless vertical velocity

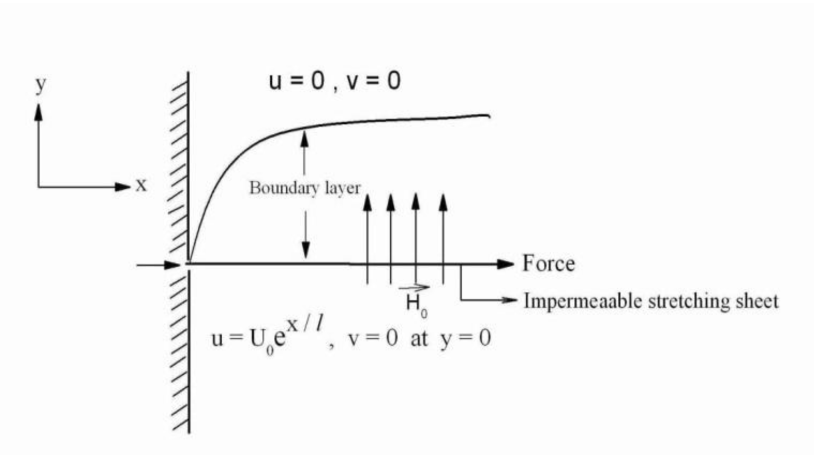

Figure 1: Schematic diagram of exponential stretching sheet.

3. MATHEMATICAL FORMULATION

We investigate the steady 2D flow over an exponentially stretching sheet of an electrically conducting, incompressible fluid of BSS. It is assumed that the boundary sheet is travelling axially at an exponentially fast rate (Figure 1). It is also assumed that the sheet is heater than the surrounding liquid, i.e.,$T_w(x) > T_\infty$. The governing boundary layer equations of continuity, heat transfer and momentum in a flow of BSS over an exponentially stretching sheet problem are as follow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}= 0\,, \tag{3.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= \upsilon\frac{\partial^2 u}{\partial y^2}-\left(\frac{\mu_m^2\,\sigma H_0^2}{\rho}\right)u-\upsilon'\frac{\partial^4 u}{\partial y^4} \tag{3.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3.3}$$

where u and v be the fluid velocity components in x and y directions respectively, μ_m is the magnetic permeability, υ is the kinematics coefficient of viscosity, σ be the electrical conductivity, ρ is the density of the fluid, υ' is couple stress viscosity, k be the fluid thermal conductivity and μ is the dynamic viscosity.

We employ the below mentioned boundary conditions on temperature and velocity:

$$u = U_w(x) = U_0\exp\left(\frac{x}{l}\right), \frac{\partial^2 u}{\partial y^2}=0, v=0$$

$$\begin{cases} T = T_w = T_\infty + (T_w - T_\infty)e^{\frac{x}{l}} \text{ in } \quad \text{PEST} \\ -k\dfrac{\partial T}{\partial y} = T_1 e^{\frac{3x}{2l}} \qquad\qquad \text{in} \quad \text{PEHF} \end{cases} \quad \text{at} \quad y=0,$$

$$u \to 0,\ \frac{\partial^2 u}{\partial y^2}\to 0, \quad T\to T_\infty \qquad \text{as} \qquad y\to\infty, \tag{3.4}$$

Where U_w be the stretching velocity of the boundary and l be the reference length.

The non−dimensionalization and transformation are used and are given below:

$$(X\,,Y) = \left(\frac{x}{l},\frac{y}{l}\right), \quad (U\,,\ V) = \left(\frac{u}{\sqrt{U_0 v}}\ ,\ \frac{v}{\sqrt{U_0 v}}\right) \tag{3. 5}$$

Using the transformation (3.5), the equations (3.1) - (3.3) becomes to

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \tag{3.6}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2}-QU - C\frac{\partial^4 U}{\partial Y^4}, \tag{3.7}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2}+\frac{vc}{C_p}\left(\frac{\partial U}{\partial Y}\right)^2, \tag{3.8}$$

where $Q = \frac{\sigma\,\mu_m^2 H_0^2}{\rho}$, $C = \frac{\gamma'}{l^2\gamma}$ and $Pr = \frac{\mu\, C_p}{k}$.

Using (3.5), the boundary conditions (3.4) reduce to

$$U = \sqrt{\frac{U_0}{\gamma}}e^X,\ \frac{\partial^2 U}{\partial Y^2}=0, V=0$$

$$\begin{cases} T = T_w = T_\infty + (T_w - T_\infty)e^X, \ \text{in} \quad \text{PEST} \\ \dfrac{\partial T}{\partial Y} = \dfrac{T_1 l}{k}e^{\frac{3}{2}X}, \qquad\qquad in \quad PEHF \end{cases} \text{at } Y=0$$

$$U\to 0,\ \frac{\partial^2 U}{\partial Y^2}\to 0, \quad T\to T_\infty \qquad \text{as } Y\to\infty. \tag{3.9}$$

The stream function ψ(X , Y) that satisfies the continuity equation (3.6), given by

$$U = \frac{\partial\psi}{\partial Y}, \qquad V = -\frac{\partial\psi}{\partial X} \tag{3.10}$$

into the equations (3.7) and (3.8), we get

$$C\frac{\partial^5\psi}{\partial Y^5}-\frac{\partial^3\psi}{\partial Y^3}+\frac{\partial\psi}{\partial Y}\frac{\partial^2\psi}{\partial X\,\partial Y}-\frac{\partial\psi}{\partial X}\frac{\partial^2\psi}{\partial Y^2}+Q\frac{\partial\psi}{\partial Y}=0, \tag{3.11}$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial T}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial T}{\partial Y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2}+\frac{\mu\gamma}{pC_p l^2}\left(\frac{\partial^2\psi}{\partial Y^2}\right)^2. \tag{3.12}$$

By means of the boundary conditions, $\psi(X, Y)$ can be found from the equation (3.9) :

$$\frac{\partial\psi}{\partial Y} = \sqrt{\frac{U_0}{Y}}e^X, \quad \frac{\partial\psi}{\partial X} = 0, \quad \frac{\partial^3\psi}{\partial Y^3} = 0,$$

$$\begin{cases} T = T_w = T_\infty + (T_w - T_\infty)e^X, & \text{in PEST} \\ \frac{\partial T}{\partial Y} = \frac{T_1 l}{k}e^{\frac{3}{2}X}, & \text{in } PEHF \end{cases} \quad \text{at } Y = 0$$

$$\frac{\partial\psi}{\partial Y} \to 0, \quad \frac{\partial^3\psi}{\partial Y^3} \to 0, \quad T \to T_\infty \quad \text{as} \quad Y \to \infty \tag{3.13}$$

We present a similarity solution for the momentum equation (3.11) in the subsequent section.

4. MOMENTUM EQUATION SOLUTION

The following similarity transformation converts the partial differential equation (2.11), into an ordinary differential equation:

$$\psi(X . Y) = \sqrt{2Re}f(\eta)\exp\left(\frac{X}{2}\right), \tag{4.1}$$

here $\eta = \left[Y\sqrt{\frac{Re}{2}}\exp\left(\frac{X}{2}\right)\right]$ be similarity variable.

Substituting equation (4.1) into the equations (3.11) and (3.13), we got the nonlinear boundary value problem presented below:

$$Cf_{\eta\eta\eta\eta\eta} - f_{\eta\eta\eta} + 2f_\eta^2 - ff_{\eta\eta} + 2Qf_\eta = 0, \tag{4.2}$$

$$f = 0, \quad f_\eta = 1, \quad f_{\eta\eta\eta} = 0 \quad \text{at} \quad \eta = 0,$$

$$f_\eta \to 0, \quad f_{\eta\eta\eta} \to 0 \quad \text{at} \quad \eta = \infty, \tag{4.3}$$

and additionally, to solve the fifth-order differential equation (4.2), we made the following assumptions:

$$f_{\eta\eta} = \alpha, \quad f_{\eta\eta\eta\eta} = \beta \quad \text{at} \quad \eta = 0, \tag{4.4}$$

where α and β are unknown initial values. So as to use the DTM to find the solution of the equation (4.2), z used as a new independent variable and is defined as $z = 1 - e^{-\eta}$.

The non - linear equation (4.2) now becomes

$$Cf''''' - 10Cf'''' - (1-25C)f''' - (15C-3)f'' - (1-C-2Q)f' - C(4z - 6z^2 + 4z^3 - z^4)f''''' + 10C(3z - 3z^2 + 4z^3)f'''' + ff' + zff'' + (1-25C)(2z - z^2)f''' - (3-15C)zf'' - ff'' + 2(f')^2 - 2z(f')^2 = 0, \tag{4.5}$$

where prime represents the differentiation with respect to z. From equations (4.3) and (4.4), in terms of z, the boundary conditions for solving equation (4.5) are obtained as:

$$f = 0, \quad f' = 1, \quad f'' = \alpha + 1, \quad f''' = 3\alpha + 2,$$

$$f'''' = 6 + 11\alpha + \beta \quad \text{at} \quad z = 0,$$

And

$$f''' = 0 \quad f' = 0 \quad \text{at} \quad z = 1. \tag{4.6}$$

The differential transform of f, $D^k\{f(z)\}$ is given by the following equation:

$$F[k] = D^k\{f(z)\} = \left[\frac{1}{k!}\frac{d^k f(z)}{dz^k}\right]_{z=0}. \tag{4.7}$$

Applying differential transform on the equation (4.5), we obtain the recurrence relation.

On employing differential transform on boundary conditions (4.6) at $z = 0$, we obtain

$$F[0] = 0, \quad F[1] = 1, \quad F[2] = \left(\frac{\alpha + 1}{2}\right),$$

$F[3] = \left(\frac{\alpha}{2} + \frac{1}{3}\right)$, $F[4] = \left(\frac{11\alpha}{24} + \frac{\beta}{24} + \frac{1}{4}\right)$. (4.8)

$F[5], F[6], F[7]$,... are estimated from the recurrence relation and boundary conditions by using Mathematica. Taking inverse differential transform on (4.7), we get a truncated series solution for $f(z)$ in the form:

$f(z) = \sum_{k=0}^{\infty} F[k]z^k \Rightarrow f(\eta) = \sum_{k=0}^{\infty} F[k](1 - e^{-\eta})^k$ (4.9)

Where α and β are obtained by means of unused conditions $f'(1) = 0$ and $f'''(1) = 0$ in the equation (4.6) for fixed value of Q and C (Table 1). The values of Q and C determine how well the aforementioned series converges.

Table 1: α and β values for different values of Q and C.

Q	C	α	β
0.0	0.10	-0.9953	4.2800
	0.15	-0.9544	3.3200
	0.20	-0.9254	2.7827
	0.30	-0.8866	2.1696
0.5	0.10	-1.1441	6.2463
	0.15	-1.0817	4.8198
	0.20	-1.0030	3.4597
	0.30	-0.9763	3.0718
1.0	0.10	-1.2733	8.1112
	0.15	-1.1944	6.2368
	0.20	-1.1371	5.1600
	0.30	-1.0584	3.9354
2.0	0.10	-1.4877	11.6072
	0.15	-1.3848	8.8913
	0.20	-1.3093	7.3352
	0.30	-1.2035	5.5649

5. HEAT TRANSFER ANALYSIS

So as to solve the equation (3.12), we took non−isothermal boundary conditions, that PEST and PEHF.

5.1. PEST

The non-dimensional temperature $\theta(\eta)$ is given as:

$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$ (5.1)

Where $T - T_\infty = \theta(\eta)e^X$ and $T_w - T_\infty = e^X$.

Using the equation (5.1) in the equation (3.12), we got the non-linear ordinary differential equation for θ(η) as given below:

$\theta_{\eta\eta} + Prf\theta_\eta - 2Prf_\eta\theta + PrE\left(f_{\eta\eta}\right)^2 = 0$ (5.2)

Equations (3.13) and (5.1) provide the boundary conditions for solving equation (5.2), which take the following form:

$\theta = 0$ as $\eta \to \infty$ and $\theta = 1$ at $\eta = 0$, (5.3)

and additionally, so as to solve the second order differential equation (5.2), we made the following assumption:

$\theta' = \gamma$ at $\eta = 0$, (5.4)

where γ is unknown initial value. In order to use the DTM for solving (5.2), we use

$$z = 1 - e^{-\eta}$$

$$z = 1 - e^{-\eta} \Rightarrow \frac{dz}{d\eta} = e^{-\eta} = 1 - z\,,$$

$$\theta_{\eta} = \frac{d\theta}{dz}\frac{dz}{d\eta} = (1-z)\theta', \tag{5.5}$$

$$\theta_{\eta\eta} = (1-z)^2\theta'' - (1-z)\theta',$$

where prime characterises the differentiation with respect to z. By means of the equation (5.5), the nonlinear equation (5.2) now becomes

$$(1-z)\theta'' - \theta' + Pr\, f\, \theta' - 2Pr\, f'\theta + PrE(1-z)\big((1-z)f'' - f'\big)^2 = 0 \tag{5.6}$$

From equations (5.3) and (5.4) in terms of z, the boundary conditions for solving equation (5.6) are found as

$$\theta = 1,\quad \theta' = \gamma \quad \text{at } z = 0 \text{ and } \theta = 0 \quad \text{at} \quad z = 1 \tag{5.7}$$

The differential transforms of f and θ are denoted by F[k] and G[k]. Applying the differential transform on the equation (5.6), we obtain the recurrence relation:

On employing the differential transform to the conditions (5.7) at $z = 0$, we get

$$G[0] = 1\,,\quad G[1] = \gamma \tag{5.8}$$

$G[2], G[3],\ G[4]$ ...are computed from the recurrence relation and boundary conditions with $F[0],\ F[1],\ F[3]$ ... .by using Mathematica. By inverse differential transform, we get a truncated series solution for θ(z) in the form:

$$\theta(z) = \sum_{k=0}^{\infty} G[k]z^k \Rightarrow \theta(\eta) = \sum_{k=0}^{\infty} G[k](1-e^{-\eta})^k \tag{5.9}$$

where γ is determined using unused condition $\theta(1) = 0$ in the equation (5.7) for fixed value of the parameters $Q,\ C,\ Pr$ and E (Table 2). The aforementioned series' convergence is depending upon Q, C, Pr, and E.

5.2. PEHF

Now the non-dimensional temperature $\phi(\eta)$ is defined as

$$\phi(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\,,$$

where

$$T - T_\infty = \frac{T_1 l}{k}\sqrt{\frac{2}{R_e}}\, e^{\frac{3X}{2}}\, \phi(\eta) \quad \text{and}$$

$$T_w - T_\infty = \frac{T_1 l}{k}\sqrt{\frac{2}{R_e}}\, e^{\frac{3X}{2}} \tag{5.10}$$

Using the equations (3.13) and (5.10) in the equation (3.12), we found the non-linear ordinary differential equation for $\phi(\eta)$ in below form:

$$\phi_{\eta\eta} + Prf\phi_\eta - 3Prf_\eta\phi + PrE\big(f_{\eta\eta}\big)^2 = 0 \tag{5.11}$$

Equations (3.13) and (5.10) yield the following boundary conditions for solving equation (5.11):

$$\phi = -1, \text{at } \eta = 0 \text{ and } \phi = 0, \text{as } \eta \to \infty \tag{5.12}$$

and additionally, so as to solve the second order differential equation (5.11), we made the following assumption:

$$\phi = \lambda \quad \text{at} \quad \eta = 0 \tag{5.13}$$

where λ is unknown initial value. So as to use the DTM for solving the equation (5.11), we use $z = 1 - e^{-\eta}$.

$$z = 1 - e^{-\eta} \Rightarrow \frac{dz}{d\eta} = e^{-\eta} = 1 - z,$$

$$\phi_\eta = \frac{d\phi}{dz}\frac{dz}{d\eta} = (1-z)\phi',$$

(5.14)

$$\phi_{\eta\eta} = (1-z)^2\phi'' - (1-z)\phi'$$

where prime denotes the differentiation with respect to z. By means of the equation (5.14), the nonlinear equation (5.11) now becomes

$$(1-z)\phi''-\phi'+\Pr f\,\phi'-3\Pr f'\phi+PrE(1-z)\left((1-z)f''-f'\right)^2=0\,. \tag{5.15}$$

Equations (5.12) and (5.13) are used to determine the boundary conditions for solving equation (5.15) in terms of z:

$$\phi=\lambda,\ \ \phi'=-1\ \text{ at }\ z=0\ \text{ and }\ \phi=0,\text{at }\ z=1. \tag{5.16}$$

The equation (5.15) is in analogy with that of PEST instance (equation (5.6) but vary only by θ and E, i.e., the dependent variable θ and the Eckert number E in PEST must be replaced by ϕ and the scaled Eckert number Es in PEHF. Applying the differential transform on the equations (5.15) and (5.16), we get the recurrence relations as described in PEST case and conditions $\Phi[0]=\lambda$, and $\Phi[1]=-1$. $L[2]$. $L[3]$, $L[4]$ … … ….are calculated using the above recurrence relations and along with conditions, where L[k] is the differential transform of ϕ. By the inverse differential transform, we get a truncated series solution for $\Phi(z)$ in the form:

$$\phi(z)=\sum_{k=0}^{\infty}L[k]z^k\Rightarrow\phi(\eta)=\sum_{k=0}^{\infty}L[k](1-e^{-\eta})^k$$
(5.17)

where λ is determined using unused condition $\phi(1)=0$ in the equation (5.16) for fixed value of Q, C, Pr and Es (Table 2). The values of Q, C, Pr, and Es determine whether the mentioned series will converge. Having discussed about the stretching sheet problems in BSS, we now present the results found in the study.

Table 2: Values of γ in PEST and λ in PEHF for Q, C, Pr and $E(Es)$ values.

Q	C	Pr	$E(E_s)$	PEST $\Theta'(0)=\gamma$	PEHF $\Phi(0)=\lambda$
1.0	0.5	1.0	1.0	-0.407177	1.15493
		1.5		-0.448022	1.04401
		2.0		-0.480362	0.983118
		2.5		-0.503915	0.944525
		3.0		-0.966957	0.917971
1.0	0.5	3.0	1.0	-1.49716	0.627298
			1.5	-1.00800	0.772635
			2.0	-0.518838	0.917971
			2.5	-0.029676	1.06331
			3.0	0.459486	1.20864
1.0	0.5	3.0	2.0	-0.518838	0.917971
1.5				-0.234599	1.00423
2.0				0.0404596	1.08992
2.5				0.305136	1.17446
3.0				0.558723	1.25740
1.0	0.1	3.0	2.0	0.554201	1.27028
	0.3			-0.160968	1.02970
	0.5			-0.518838	0.917971

6. RESULTS AND DISCUSSION

In the current study, we studied the heat transfer and hydromagnetic boundary layer flow characteristics of a BSS over an exponentially stretching impermeable sheet. The equations were transformed into non-linear ordinary differential equations by applying the proper local similarity transformation. The flow phenomena are described by nonlinear partial differential equations. The solutions of transformed equations are obtained by means of DTM. The attention has been focused on the variations of Q, C, Pr and $E(Es)$ in both PEST and PEHF. The results of the work are showed in the form of graphs.

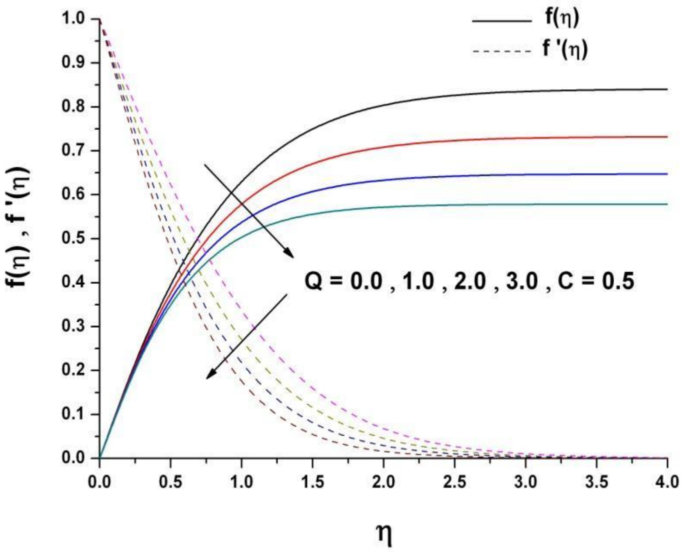

Figure 2: Plots of f(η) and $f^0(\eta)$ versus η for C = 0.5 and different values of Q.

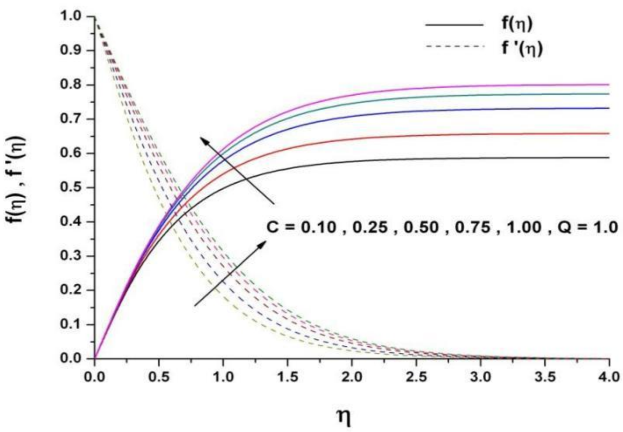

Figure 3: Plots of f(η) and $f^0(\eta)$ versus η for Q = 1.0 and different values of C.

In Figure (2) and (3), the graphs of $f(\eta)$ and $f'(\eta)$ versus η are plotted for chosen values of the Q and C.

Any motion in the liquid is opposed by the magnetic field's effect. Based on this reason, we find from Figure (2) that the effect of increasing Q is to decrease the velocity profiles through the boundary layer. The applied magnetic field causes the velocity boundary layer to contract transversely, and this causes the Lorentz force to significantly oppose motion.

In Figure (3), the plots of $f(\eta)$ and $f'(\eta)$ versus η are plotted for chosen values of C with Q=1.0. It has been found that increasing C has the effect of increasing the velocity through the boundary layer. The reason is that the suspended particle in the BSS rise in the carrier liquid velocity and henceforth to increase the flow.

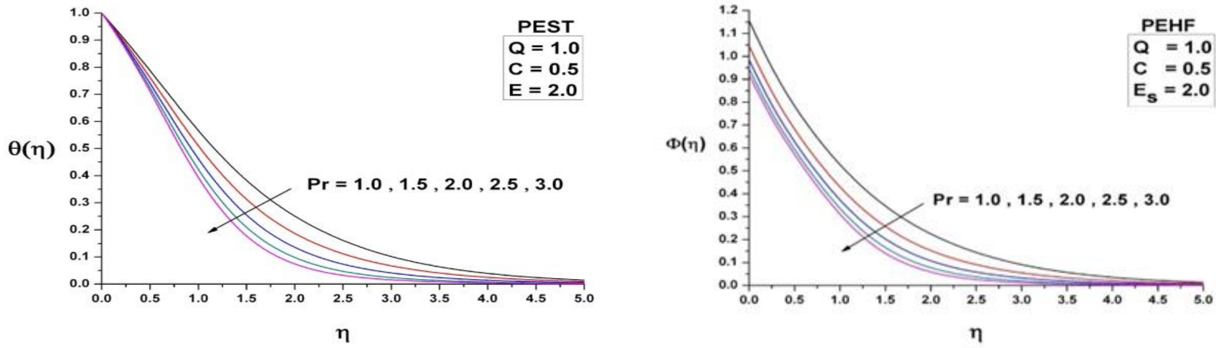

Figure 4: Plots of θ(η) and Φ(η) in PEST and PEHF cases for different values of Pr.

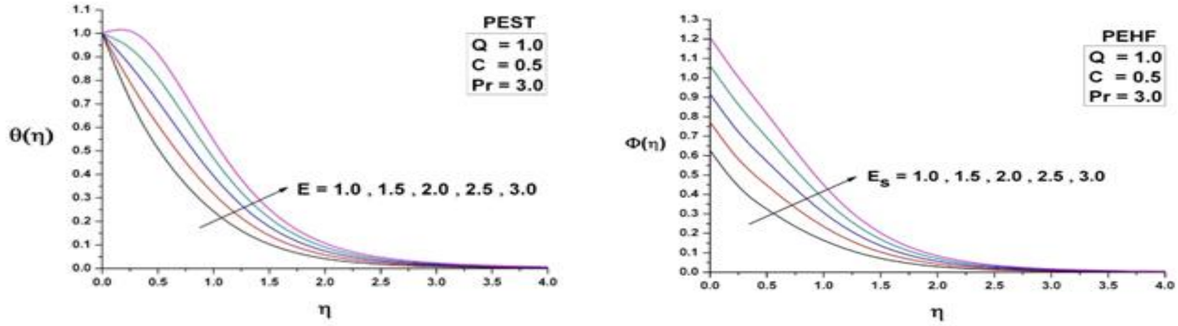

Figure 5: Plots of θ(η) and Φ(η) in PEST and PEHF cases for different values E(Es).

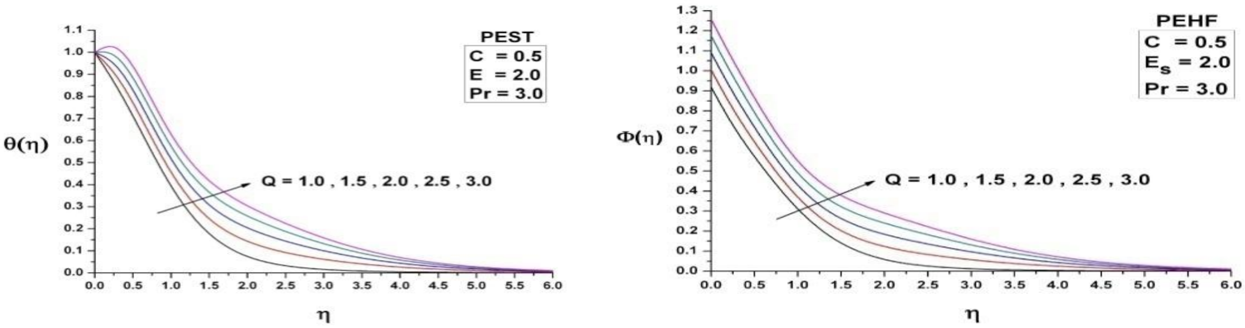

Figure 6: Plots of θ(η) and Φ(η) in PEST and PEHF cases for different values of Q.

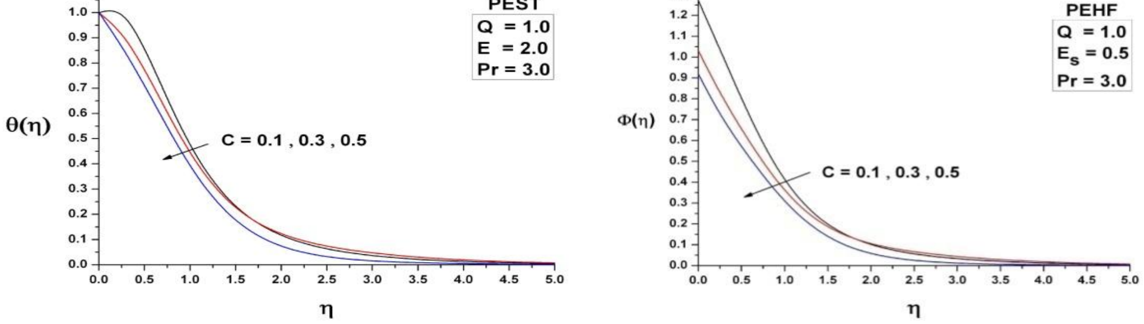

Figure 7: Plots of θ(η) and Φ(η) in PEST and PEHF cases for different values of C of E(Es).

In Figure (4) - (7), The plots of $\theta(\eta)$ and $\phi(\eta)$ verses η are drawn for chosen values of Pr, $E(Es)$, Q, and C in both PEST and PEHF cases. It is found that in PEST cases the wall temperature distribution is at unity, while it may be unity or other than unity in the PEHF cases on the wall.

Figure (4) elucidates the influence of Pr on temperature profile $\theta(\eta)$ and $\phi(\eta)$ in PEST and PEHF cases. From these graphs it is observed that the increasing value of Pr is decreases the temperature. This is due the large values of Pr result in thinning the thermal boundary layer. Figure (5) shows the effect of Eckert number $E(Es)$ on temperature profiles in PEHF and PEST cases. The graphs reveal the effect of increasing E enhances the temperature in the flow region. This is because of the fact that the heat energy stored in the considered liquid because of frictional heating. Figure (6) illustrates how raising Q causes the temperature to rise. The graphs in equation (7) show that a rise in C values causes the temperature to decrease. We now present the conclusion of the above study.

7. CONCLUSION

Some of the important findings of this chapter pertaining to exponentially stretching sheet are listed below:

1. The magnetic field has the effect of giving the electrically conducting liquid stiffness.
2. Thermal boundary layer thickness increases and momentum boundary layer thickness decreases with increasing values of Chandrasekhar number Q.
3. Thermal boundary layer thickness decreases and momentum boundary layer thickness increases with an increase in couple stress parameter C.
4. The temperature is going to decrease as the Prandtl number (Pr) increases.

The increase in the Eckert number E(Es) is to increase the temperature.

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